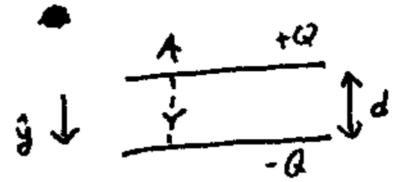
# Huang Fall 07 Final - Solutions

1. · Capacitance of a parallel plate capacitor of area A, separation d, empty:



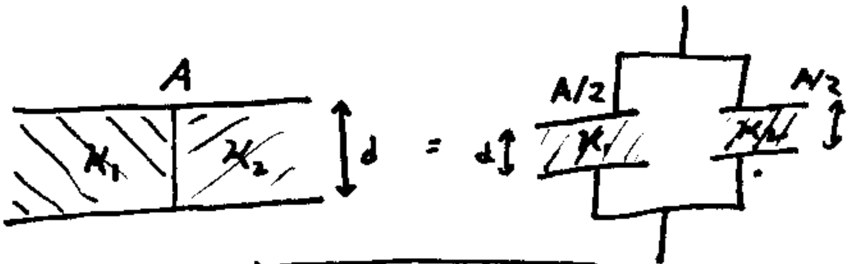

- Put charge +Q on top plate, −Q on bottom
- Field between plates is $\vec{E} = \underset{\text{top plate}}{\frac{\sigma}{2\epsilon_0}(\widehat{down})} + \underset{\text{bottom plate}}{\frac{-\sigma}{2\epsilon_0}(\widehat{up})}$
- $\sigma = \frac{Q}{A}$ ⇒ $\vec{E} = \frac{Q}{A\epsilon_0}(\widehat{down})$

- to find $\Delta V$, use path in dotted line:

$$\Delta V = -\int \underset{=d\hat{y}}{\vec{E}\cdot d\vec{s}} = -\int_{y=0}^{y=d} \left(\frac{Q}{A\epsilon_0}\right)\hat{y}\cdot\hat{y} = \frac{-Qd}{A\epsilon_0}$$

- $\cancel{C = \frac{|\Delta V|}{|Q|}}$ · $C = \frac{|Q|}{|\Delta V|} = \frac{Q}{\frac{Qd}{A\epsilon_0}} = \frac{A\epsilon_0}{d}$

a)



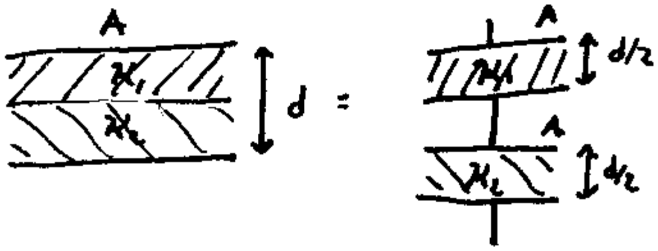

⇒ $C = \kappa_1 \cdot \frac{(A/2)\epsilon_0}{d} + \kappa_2 \frac{(A/2)\epsilon_0}{d}$

$C_{diel} = \kappa C_0$

adding in parallel!

$$\Rightarrow \boxed{C = \frac{\kappa_1 + \kappa_2}{2} \frac{\epsilon_0 A}{d}}$$

b)



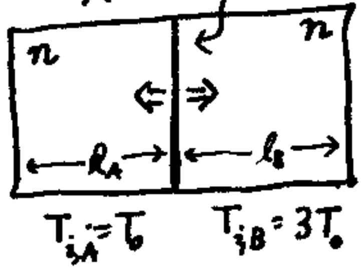

$$\Rightarrow \frac{1}{C} = \frac{1}{\kappa_1 \frac{A\epsilon_0}{(d/2)}} + \frac{1}{\kappa_2 \frac{A\epsilon_0}{(d/2)}} = \frac{1}{\left(\frac{A\epsilon_0}{d}\right)}\left(\frac{1}{2\kappa_1} + \frac{1}{2\kappa_2}\right)$$

adding in series!

$$\Rightarrow \cancel{C = \frac{1}{2}\frac{\kappa_1\kappa_2}{\kappa_1+\kappa_2}\frac{\epsilon_0 A}{d}} \qquad \boxed{C = \frac{2\kappa_1\kappa_2}{\kappa_1+\kappa_2}\frac{\epsilon_0 A}{d}}$$

2.

B piston, free to move,




Mechanical Equilibrium ⇒ all forces cancel,

$$\Rightarrow P_A = P_B$$

a) $$\Delta Q_{total} = 0 = m_A c_A \Delta T_A + m_B c_B \Delta T_B$$

But both sides A & B have the same mass & are of the same type ⇒

$$m_A c_A = m_B c_B$$

$$\Rightarrow \Delta T_A + \Delta T_B = 0 \Rightarrow (T_f - T_0) + (T_f - 3T_0) = 0$$

$$\Rightarrow 2T_f = 4T_0 \Rightarrow \boxed{T_f = 2T_0}$$

b) Initially: $P_A = P_B \Rightarrow \frac{\cancel{n_A R} T_A}{V_A} = \frac{\cancel{n_B R} T_B}{V_B} \Rightarrow$

$$\Rightarrow \frac{V_A}{V_B} = r_i = \frac{T_{A,0}}{T_{B,0}} = \frac{T_0}{3T_0} \Rightarrow \boxed{r_i = \frac{1}{3}}$$

c) Finally: $$r_i = \frac{T_{A,f}}{T_{B,f}} = \frac{2T_0}{2T_0} \Rightarrow \boxed{r_i = 1}$$

d) For the reversible path, separate A & isobarically expand from temp $T_0$ to $2T_0$

$$dU = dQ - dW \Rightarrow dQ = dU + dW = \frac{d}{2} nR\,dT + P\,dV$$

$$= \frac{d}{2} nR\,dT + \cancel{P} \frac{nR\,dT}{\cancel{P}}$$

$$V = \frac{nkT}{P}$$

$$\Rightarrow dQ = \frac{d+2}{2} nR\,dT$$

$$\Rightarrow \Delta S_A = \int_{T_0}^{2T_0} \frac{dQ}{T} = \int_{T_0}^{2T_0} \frac{d+2}{2} nR \frac{dT}{T} = \frac{d+2}{2} nR \ln \frac{2T_0}{T_0}$$

$$\Rightarrow \Delta S_A = \left(\frac{d+2}{2} \ln 2\right) nR$$

monatomic $\Rightarrow d=3$

$$\boxed{\Rightarrow \Delta S_A = \frac{5 \ln 2}{2} nR}$$

e) Reversible Process: Separate B & isobarically contract from $T=3T_0$ to $T=2T_0$

$$\Rightarrow \Delta S_B = \int_{3T_0}^{2T_0} \frac{dQ}{T} = \frac{d+2}{2} nR \ln \frac{2T_0}{3T_0} = -\frac{d+2}{2} \ln \frac{3}{2} nR$$

$$\Rightarrow \boxed{\Delta S_B = -\left(\frac{5 \ln 3/2}{2}\right) nR}$$

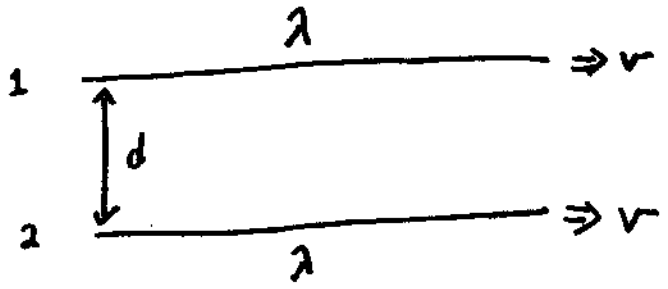
f) No heat exchanged w/ environment

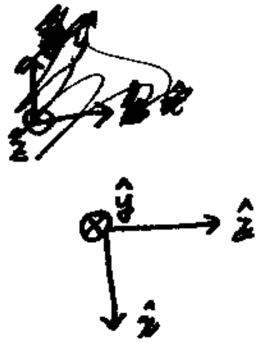
$$\Rightarrow \Delta S = \Delta S_A + \Delta S_B = \frac{5}{2} \ln 2 \, nR - \frac{5}{2} \ln \frac{3}{2} nR$$

$$= \left[\frac{5}{2} nR\right]\left(\ln 2 - \ln \frac{3}{2}\right)$$

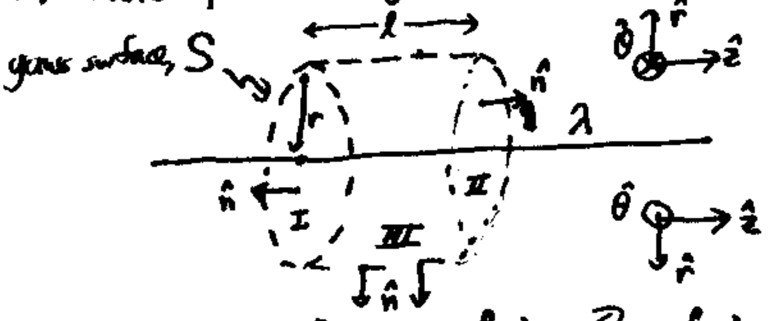
$$\boxed{\Delta S = \frac{5}{2} \ln \frac{4}{3} nR > 0}$$

3.




a) Field produced by charge of wire 1: Wire has cylindrical symmetry ⇒ use Gauss' Law!




By symmetry, $\vec{E} = E_r(\vec{r})\hat{r}$ — only points radially away; only depends on radial distance from wire

$$\Phi_S = \Phi_I + \Phi_{II} + \Phi_{III} = \cancel{\int_I \vec{E}\cdot(-\hat{z})dA} + \cancel{\int_{II} \vec{E}\cdot(+\hat{z})dA} + \int_{III} \vec{E}\cdot\hat{r}\,dA$$

$\Phi_I$: normal vector $-\hat{z}$; $\Phi_{II}$: normal vector $+\hat{z}$; $\Phi_{III}$: normal vector $\hat{r}$

$\vec{E}\cdot\hat{z} \propto \hat{r}\cdot\hat{z} = 0$

$$= \int_{III} E(r)\,\hat{r}\cdot\hat{r}\,dA = \int_{III} E(r)\,dA$$

But $r = $ const. on III, so $\Phi = E(r)\int_{III} dA = E(r)A_{III} = 2\pi r l E(r)$

Gauss: $\Phi = \frac{Q_{enc}}{\epsilon_0} \Rightarrow 2\pi r l E(r) = \frac{\lambda l}{\epsilon_0} \Rightarrow \vec{E}(r) = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$

$$\Rightarrow \boxed{\vec{E}_{@2 \text{ from } 1} = \frac{\lambda}{2\pi\epsilon_0 d}\hat{x}}$$

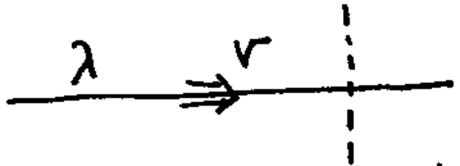
r = d; @ 2, $\hat{r} = \hat{x}$

b) $\vec{F}_E = q\vec{E} \Rightarrow \vec{f}_E = \frac{q\vec{E}}{l} = \lambda\vec{E}$ ⇒

force per unit length

$$\boxed{\vec{f}_{E \text{ on } 2 \text{ by } 1} = \frac{\lambda^2}{2\pi\epsilon_0 d}\hat{x}}$$

c)




$I = \frac{\Delta q}{\Delta t}$ = how much charge passes through dashed line per unit time?

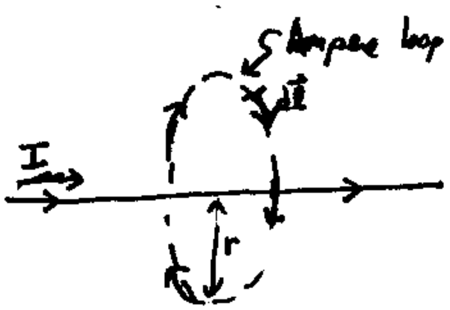
or: $I = \frac{dq}{dt} = \frac{dq}{dx}\frac{dx}{dt} = \lambda v$

in the $\Delta t$, a length $v\Delta t$ passes

$\Rightarrow$ a charge $\lambda v \Delta t$ passes $\Rightarrow I = \frac{\Delta q}{\Delta t} = \frac{\lambda v \Delta t}{\Delta t} = \lambda v$

$\Rightarrow \boxed{I = \lambda v \text{ to the right}}$

d)




cyl. sym $\Rightarrow$ use Ampere!

By sym, $\vec{B}$-field has to depend only on $r$ &, by the RHR, should point only in the $\hat{\theta}$-direction

$\Rightarrow \vec{B} = B(r)\hat{\theta}$ Ampere: $\oint_{\ell} \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$

$d\vec{\ell} = r\,d\theta\,\hat{\theta}$

$\Rightarrow \int_0^{2\pi} \left(B(r)\hat{\theta}\right)\left(r\,d\theta\,\hat{\theta}\right) = \mu_0 I$

$= rB(r)\int_0^{2\pi} d\theta = \mu_0 I \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta}$

r = const along loop!

@ 2, $r = d$, $\hat{\theta} = +\hat{y}$ (into the page), $I_1 = \lambda v$

$\Rightarrow \boxed{\vec{B}_{@2 \text{ from } 1} = \frac{+\mu_0 \lambda v}{2\pi d}\hat{y}}$

~~#~~ e) $\vec{F}_B = q\vec{v}\times\vec{B}$ $\Rightarrow$ $\vec{f}_B = \lambda\vec{v}\times\vec{B}$

force per unit length

charge per unit length

$\vec{v} = v\hat{z}$ $\quad$ $\vec{B} = \frac{\mu_0\lambda v}{2\pi d}(-\hat{y})$

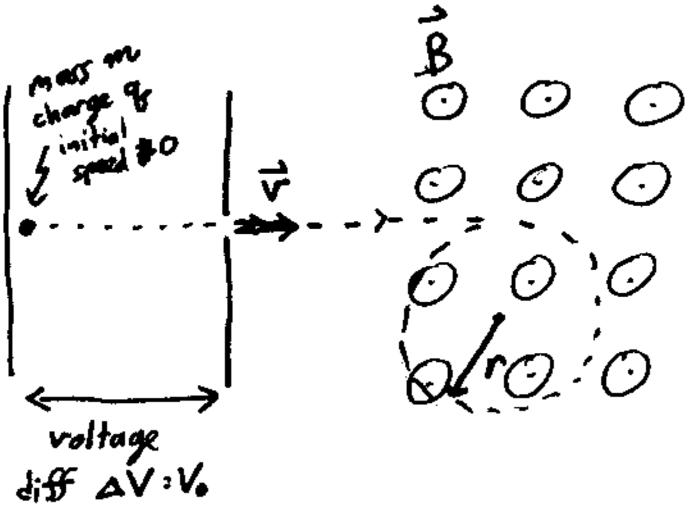
$$\vec{f}_B = \lambda v\frac{\mu_0\lambda v}{2\pi d}\underbrace{(\hat{z}\times(-\hat{y}))}_{-\hat{x}} \Rightarrow \boxed{\vec{f}_{B\text{ on }2\text{ by }1} = \frac{\mu_0\lambda^2 v^2}{2\pi d}(-\hat{x})}$$

f) $|\vec{f}_E| = |\vec{f}_B| \Rightarrow \frac{\cancel{\lambda^2}}{\cancel{2\pi}\epsilon_0\cancel{d}} = \frac{\mu_0\cancel{\lambda^2}v^2}{\cancel{2\pi d}} \Rightarrow v^2 = \frac{1}{\epsilon_0\mu_0}$

$$\Rightarrow \boxed{v = \frac{1}{\sqrt{\epsilon_0\mu_0}} = c}$$

speed of light!

4.




a) $v$: conservation of energy: $\Delta U = q\Delta V$

$$\frac{1}{2}mv^2 - \cancel{\frac{1}{2}m\cdot 0^2} = qV_0 \Rightarrow v = \sqrt{\frac{2qV_0}{m}}$$

in B-field, $|\vec{F}| = |q\vec{v}\times\vec{B}| = \underset{\vec{v}\perp\vec{B}}{qvB}$

central force: $|F| = \frac{mv^2}{r} \Rightarrow qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$

$$\Rightarrow r = \frac{m}{q}\sqrt{\frac{2qV_0}{m}}\,\frac{1}{B} \Rightarrow \boxed{r = \sqrt{\frac{2mV_0}{qB^2}}}$$

doubly charged helium: $m = 6.6\times10^{-27}\,kg$, $q = +2e$, $e =$

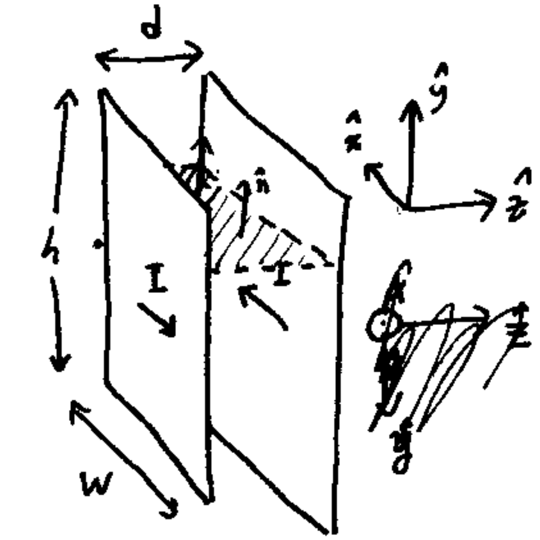
$V_0 = 2100\,V$, $B = 0.340\,T$

$$\Rightarrow \boxed{r = 2.74\,cm}$$

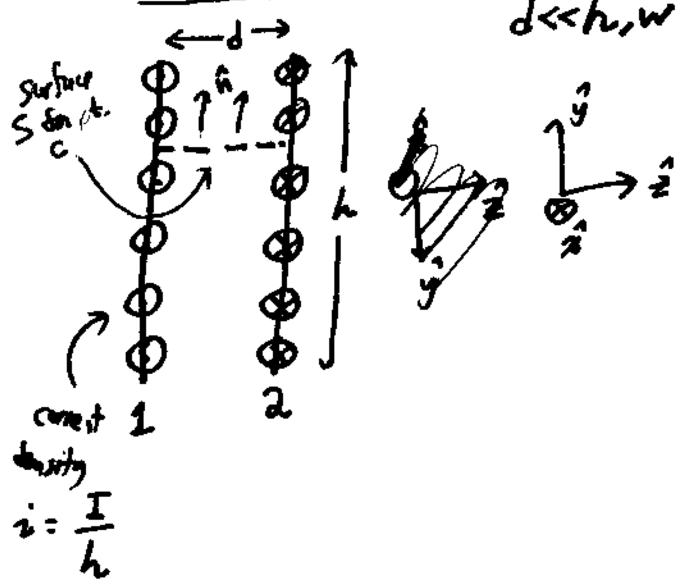
b) $T = \frac{2\pi r}{v} = \frac{2\pi m\cancel{v}}{qB}/\cancel{v} = \frac{2\pi m}{qB}$

$$\boxed{T = 2\pi\frac{m}{qB} = 3.8\times10^{-7}\,s}$$

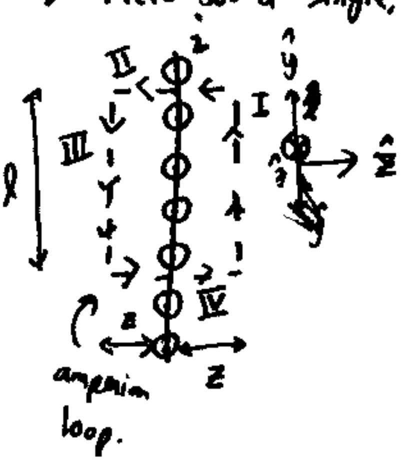
5.




Front view



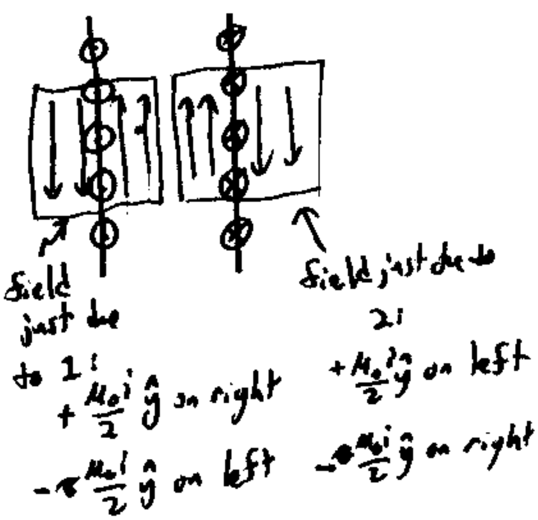

a) Field for a single, infinite plate




By sym, B depends only on z, is the same magnitude & opp direction on opposite sides of the sheet, & points in the $\pm\hat{y}$ direction (can see using RHR)

$$\Rightarrow \oint_{l} \vec{B}\cdot d\vec{l} = \int_{I} B(z)\hat{y}\cdot(+\hat{y}\,dy) + \cancel{\int_{II} B(z)\hat{y}\cdot(-\hat{z}\,dz)} + \int_{III} B(z)\hat{y}\cdot(-\hat{y}\,dy) + \cancel{\int_{IV} B(z)\hat{y}\cdot(\hat{z}\,dz)}$$

$$= +B(+z)l - B(-z)l = +2lB(+z) = \mu_0 I_{enc} = \mu_0 l i \Rightarrow \underline{B(z) = \pm\frac{\mu_0 i}{2}\hat{y}}$$




a) $\vec{B}_{between} = \vec{B}_1 + \vec{B}_2 = \underset{\text{right of 1}}{+\frac{\mu_0 i}{2}\hat{y}} + \underset{\text{left of 2}}{\frac{+\mu_0 i}{2}\hat{y}}$

$$\Rightarrow \boxed{\vec{B}_{between} = +\mu_0 i\,\hat{y}}$$

b) $\boxed{\vec{B}_{outside} = 0}$

to left of both: $-\frac{\mu_0 i}{2}\hat{y} + \frac{\mu_0 i}{2}\hat{y} = 0$

& right " " : $+\frac{\mu_0 i}{2}\hat{y} + \frac{-\mu_0 i}{2}\hat{y} = 0$

c) $$\Phi_{B,S} = \int \vec{B}\cdot d\vec{A} = \int (\mu_0 i \hat{y})\cdot(\hat{n}\,dA)$$

As shown above, we have chosen $\hat{n}$ s.t. $\hat{n} = +\hat{y}$

$$\Rightarrow \Phi_B = \int \mu_0 i\,dA(\hat{y}\cdot\hat{y}) = \mu_0 i A = \mu_0 i (d\cdot w)$$

$$\Rightarrow \boxed{\Phi_B = \mu_0 i w d}$$

d) $$\mathcal{E} = -\frac{d\Phi}{dt} = -\mu_0 w d \frac{di}{dt} = -L\frac{dI}{dt}$$

$$\Rightarrow \cancel{L = \mu_0 w d} \quad L\frac{dI}{dt} = \mu_0 w d \frac{di}{dt}$$

$$i = \frac{I}{h} \Rightarrow \frac{di}{dt} = \frac{1}{h}\frac{dI}{dt} \Rightarrow L\frac{dI}{dt} = \mu_0 w d \frac{1}{h}\frac{dI}{dt}$$

$$\Rightarrow \boxed{L = \frac{\mu_0 w d}{h}}$$