Stat 134 MT Solutions

1a). (version 1: 4 green, 3 red, 2 blue). (other version has 3 blue)

$P(X=2 \text{ or } Y=2) = P(X=2)+P(Y=2)-P(X=2,\,Y=2)$

two ways 1) $\binom{4}{2}\left(\frac{4}{9}\right)\left(\frac{3}{8}\right)\left(\frac{5}{7}\right)\left(\frac{4}{6}\right)+\binom{4}{2}\left(\frac{3}{9}\right)\left(\frac{2}{8}\right)\left(\frac{6}{7}\right)\left(\frac{5}{6}\right)-\binom{4}{2}\left(\frac{4}{9}\right)\left(\frac{3}{8}\right)\left(\frac{3}{7}\right)\left(\frac{2}{6}\right)$

G G N N   R R N N   G G R R

or 2) $\dfrac{\binom{4}{2}\binom{5}{2}+\binom{3}{2}\binom{6}{2}+\binom{4}{2}\binom{3}{2}\binom{2}{0}}{\binom{9}{4}}$

b) $P(M=2) = P(X=2, Y=2)$ done as last term in a), $=\frac{1}{7}$

$P(M=0) = P(X=0)+P(Y=0)$ (notice they can't both be 0)

$= \dfrac{\binom{4}{0}\binom{5}{4}}{\binom{9}{4}} + \dfrac{\binom{6}{4}\binom{3}{0}}{\binom{9}{4}} = \dfrac{20}{126} = \dfrac{10}{63}$

OR $= \left(\frac{5}{9}\right)\left(\frac{4}{8}\right)\left(\frac{3}{7}\right)\left(\frac{2}{6}\right)+\left(\frac{6}{9}\right)\left(\frac{5}{8}\right)\left(\frac{4}{7}\right)\left(\frac{3}{6}\right) = \frac{10}{63}$

$P(M=1) = 1-\frac{1}{7}-\frac{10}{63} = \frac{44}{63}$

$E(M) = 1\left(\frac{44}{63}\right)+2\left(\frac{9}{63}\right) = \frac{62}{63} \approx 0.984$

c) $\text{var}(M) = E(M^2) - E(M)^2$

$E(M^2) = \frac{44}{63}+4\left(\frac{1}{7}\right)^2 = \frac{80}{63}$

$\text{var}(M) = \frac{80}{63} - \left(\frac{10}{63}\right)^2 = \frac{4940}{(63)^2} \approx 0.301$

d) $P(X=2, Y=1, Z=1) = \dfrac{\binom{4}{2}\binom{3}{1}\binom{2}{1}}{\binom{9}{4}} = \dfrac{2}{7}$

(e) $A = 1^{st}\ R$, $B =$ same color

$P(A) = \frac{3}{9} = \frac{1}{3}$

$P(B) = \dfrac{\binom{4}{2} + \binom{3}{2} + \binom{2}{2}}{\binom{9}{2}} = \frac{5}{18}$ OR $= \left(\frac{4}{9}\right)\left(\frac{3}{8}\right) + \left(\frac{3}{9}\right)\left(\frac{2}{8}\right) + \left(\frac{2}{9}\right)\left(\frac{1}{8}\right) = \frac{20}{72} = \frac{5}{18}$

$P(AB) = \dfrac{\binom{3}{2}}{\binom{9}{2}} = \left(\frac{3}{9}\right)\left(\frac{2}{8}\right) = \frac{1}{12} \neq P(A)\,P(B) = \left(\frac{1}{3}\right)\left(\frac{5}{18}\right) = \frac{5}{54}$ so not indep.

OR. $P(B|A) = \frac{2}{8} \neq P(B) = \frac{5}{18}$

OR $P(A|B) = \dfrac{1/12}{5/18} = \frac{3}{10} \neq P(A) = \frac{1}{3}$

(f) binomial $(100, \frac{1}{3})$

$\mu = np = 100\left(\frac{1}{3}\right) = 33.33$

$\sigma = \sqrt{npq} = \sqrt{100\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)} = 4.71$

$P(36 \text{ to } 100 \text{ successes}) = \Phi\left(\dfrac{100+0.5-33.33}{4.71}\right) - \Phi\left(\dfrac{36-0.5-33.33}{4.71}\right)$

$= \Phi(14.26) - \Phi(0.46)$

$= 1 - 0.6772 = \boxed{0.3228}$

(version 2: 9 die rolls) (other version has 10 die rolls).

2a) $$1-\frac{\left(\frac{5}{6}\right)^3\binom{6}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4}{\binom{9}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^7} = 1-\frac{\binom{6}{2}}{\binom{9}{2}} = \frac{7}{12}$$

OR $$1-\frac{\binom{2}{0}\binom{7}{3}}{\binom{9}{3}}$$ OR $$\frac{\binom{3}{1}\binom{6}{1}+\binom{3}{2}\binom{6}{0}}{\binom{9}{2}}$$ OR $$\frac{\binom{2}{1}\binom{7}{2}+\binom{2}{2}\binom{7}{1}}{\binom{9}{3}}$$

b) $$P(A|B) = 1-P(A^c|B)$$
$$= 1-\frac{P(A^c B)}{P(B)} = 1-\frac{P(A^c)P(B|A^c)}{1-P(B^c)}$$
$$= 1-\frac{P(A^c)(1-P(B^c|A^c))}{1-P(B^c)}$$
$$= 1-\frac{\left(\frac{5}{6}\right)^3\left(1-\left(\frac{5}{6}\right)^6-6\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^5\right)}{1-\left(\frac{5}{6}\right)^9-9\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^9}$$

OR $$P(A|B) = \frac{P(AB)}{P(B)}$$
$$= \frac{P(A)-P(AB^c)}{P(B)}$$
$$= \frac{\left(1-\left(\frac{5}{6}\right)^3\right)-\binom{3}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^9}{P(B)}$$

P(1 in 3, ≥1 in next 6)

OR $$P(A|B) = \frac{P(1,1+)+P(2 \text{ in } 3)+P(3 \text{ in } 3)}{P(B)}$$
$$= \frac{\binom{3}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2\left(1-\left(\frac{5}{6}\right)^6\right)+\binom{3}{2}\left(\frac{1}{6}\right)^2\frac{5}{6}+\left(\frac{1}{6}\right)^3}{P(B)}$$

3. a) $\frac{1}{(n-1)!}\left(\frac{n-1}{n}\right) = \frac{n-1}{n!} = \frac{1}{(n-2)!\,n}$

b) $E(X) = \sum_{n=2}^{\infty} n \frac{1}{(n-2)!\,n} = \sum_{n=2}^{\infty} \frac{1}{(n-2)!} = \sum_{n=0}^{\infty} \frac{1}{n!} = e$

c) $E(X(X-2)) = \sum_{n=3}^{\infty} n(n-2)\frac{1}{(n-2)!\,n} = \sum_{n=3}^{\infty} \frac{1}{(n-3)!} = \sum_{n=0}^{\infty} \frac{1}{n!} = e$

$E(X(X-2)) = E(X^2-2X) = E(X^2) - E(2X) = e$

So $E(X^2) = 3e$

$\text{var}(X) = 3e - e^2$

<u>OR</u> $E(X^2) = \sum_{n=2}^{\infty} n^2 \frac{1}{(n-2)!\,n} = \sum_{n=2}^{\infty} \frac{n}{(n-2)!}$

$= \sum_{n=2}^{\infty} \frac{n-2}{(n-2)!} + \frac{2}{(n-2)!}$

$= \sum_{n=3}^{\infty} \frac{1}{(n-3)!} + \sum_{n=2}^{\infty} \frac{2}{(n-2)!}$

$= e + 2e = 3e$

$\text{var}(X) = 3e - e^2$