

Midterm 2 Ma 53 Spr '08

Prof



1. z is a function of x and y defined

implicitly by $z^3 + z^2x + zy^2 + x^2y = 4$.

Compute $z_x(1, 1)$ and $z_y(1, 1)$ assuming

$$z(1, 1) = 1.$$

$$3z^2z_x + 2zz_xx + z^2 + z_xy^2 + 2xy = 0$$

$$\text{@ } (1, 1)$$

$$3z_x + 2z_x + 1 + z_x + 2 = 0 \Rightarrow z_x = -\frac{1}{2}$$

$$3z^2z_y + 2zz_yx + z_yy^2 + z2y + x^2 = 0$$

$$\text{@ } (1, 1)$$

$$3z_y + 2z_y + z_y + 2 + 1 = 0 \quad z_y = -\frac{1}{2}$$



2. $x(t)$ satisfies $x'(t) = u(x(t), t)$,

where $u(x, t)$ is a given function

with $u(1, 1) = 1$, $u_t(1, 1) = 1$, $u_x(1, 1) = 1$,

and $x(1) = 1$. Compute $x''(1)$.

$$x''(t) = u_t(x(t), t) + u_x(x(t), t) x'(t)$$

$$= (u_t + u u_x)(x(t), t)$$

$$\text{(a) } t=1, \quad x''(1) = 1 + 1 \cdot 1 = 2.$$

3. a) Compute ∇f for $f(x, y, z) =$

$$z^3 + z^2x + zy^2 + x^2y \quad \text{at } (x, y, z) =$$

$$(1, 1, 1).$$

$$\nabla f = (z^2 + 2xy) \underline{i} + (2zy + x^2) \underline{j} +$$

$$(3z^2 + 2zx + y^2) \underline{k} =$$

$$6 \underline{i} + 3 \underline{j} + 6 \underline{k}$$

$$3 \underline{i} + 3 \underline{j} + 6 \underline{k}$$

b) What is the equation of the

tangent plane to the surface $f(x, y, z) = 4$

at $(x, y, z) = (1, 1, 1)$?

$$(3\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}) \cdot ((x-1)\mathbf{i} + (y-1)\mathbf{j} + (z-1)\mathbf{k}) = 0$$

$$3(x-1) + 3(y-1) + 6(z-1) = 0$$

c) Compute the linearized approximation to $f(1.01, 1.02, 1.005)$.

$$f(1.01, 1.02, 1.005) \approx$$

$$f(1, 1, 1) + f_x(1, 1, 1)(.01) + \\ f_y(1, 1, 1)(.02) + \\ f_z(1, 1, 1)(.005) =$$

$$4 + 3(.01) + 3(.02) + 6(.005) = 4.12$$

.03 .06 .03

4. Compute $\iint_D e^{-x} e^{-y} dA$,

where D is the region bounded by the line $x + y = s = \text{positive constant}$, and the x and y axes.

$$\int_0^s e^{-x} \left(\int_0^{s-x} e^{-y} dy \right) dx =$$

$$\int_0^s e^{-x} (1 - e^{x-s}) dx =$$

$$\int_0^s (e^{-x} - e^{-s}) dx = 1 - e^{-s} - se^{-s}$$

5. Compute $\iiint_E \frac{dV}{\sqrt{x^2 + y^2 + (r-z)^2}}$

where E is the "ball" $0 < \rho < R$ and

r is a constant, $r > R$.

$$2\pi \int_0^R \int_0^\pi \frac{\rho^2 \sin \varphi}{\sqrt{\rho^2 \sin^2 \varphi + (r - \rho \cos \varphi)^2}} d\varphi d\rho =$$

$$\rho^2 + \rho^2 - 2r\rho \cos \varphi$$

$$2\pi \int_0^R \frac{\rho^2}{2r\rho} \left[\sqrt{r^2 + \rho^2 - 2r\rho \cos \varphi} \right]_0^\pi d\rho =$$

$$\frac{\pi}{r} \int_0^R \rho \{ r + \rho - r + \rho \} d\rho =$$

$$\frac{2\pi}{r} \int_0^R \rho^2 d\rho = \frac{2\pi}{3r} R^3$$