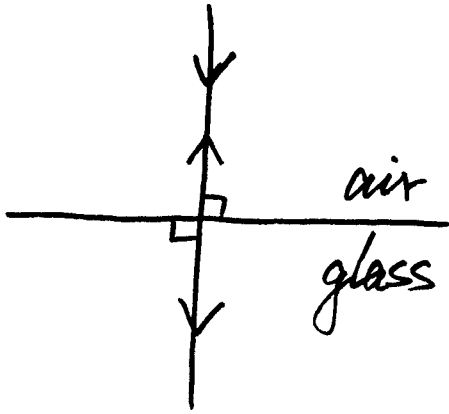


Problem 1.

Solution

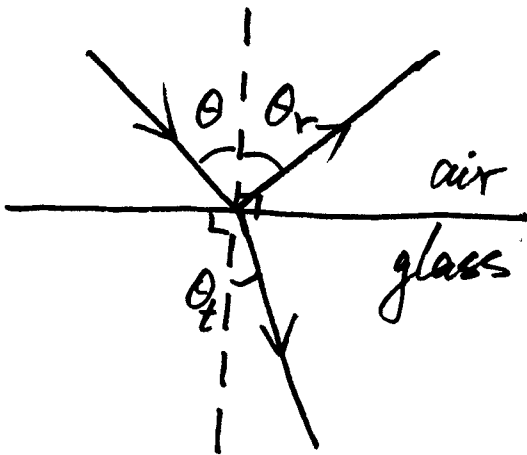
(a)

$$\theta = 0^\circ$$



reflected direction: $\theta_r = 0^\circ$
transmitted direction: $\theta_t = 0^\circ$

$$\theta = 45^\circ$$



reflected direction: $\theta_r = 45^\circ$

transmitted direction: ~~$\theta_t = 45^\circ$~~

$$\theta_t = \arcsin \frac{\sqrt{2}}{3}$$

$$n_{\text{air}} \sin \theta = n_{\text{glass}} \sin \theta_t$$

$$1 \times \sin 45^\circ = 1.5 \times \sin \theta_t$$

$$\sin \theta_t = \frac{2}{3} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{3}$$

(b) TIR is Not possible for light incident from the air side.

(c) Yes. TIR is possible for light incident from the glass side.

$$n_{\text{glass}} \sin \theta \geq n_{\text{air}} = 1$$

$$\therefore \sin \theta \geq \frac{1}{n_{\text{glass}}} = \frac{1}{1.5} = \frac{2}{3}$$

$$\theta \geq \arcsin \frac{2}{3}, \quad \text{of course } \theta \leq 90^\circ$$

(d) Light travels faster on the air side.

$$v_{\text{air}} = \frac{c}{n_{\text{air}}} = \frac{c}{1} = c$$

$$v_{\text{glass}} = \frac{c}{n_{\text{glass}}} = \frac{c}{1.5} = \frac{2}{3}c$$

Rubric

(a) no reflected ray in one picture -1.

$$\theta = 0^\circ \begin{cases} \theta_r = 0^\circ \\ \theta_t = 0^\circ \end{cases} \quad \theta_r, \theta_t \quad 1 \text{ point respectively}$$

$$\theta = 45^\circ \begin{cases} \theta_r = 45^\circ & 1 \\ \theta_t = \arcsin \frac{\sqrt{2}}{3} & \begin{array}{l} \nearrow \text{Snell law} \\ \uparrow \text{value} \end{array} \end{cases}$$

2 = 1 + 1

qualitative describe will also get the credits.

(b) 5

$$(c) \quad \theta_c = \arcsin \frac{2}{3} \quad 4.$$

$$\theta \geq \theta_c \quad 1 \quad (\theta > \theta_c \text{ cannot get this point})$$

(d) air side faster 1

$$v = \frac{c}{n} \quad 2.$$

values 1+1

Problem 2

Solution

(a)

$$\Delta X = \lambda \frac{L}{d}$$

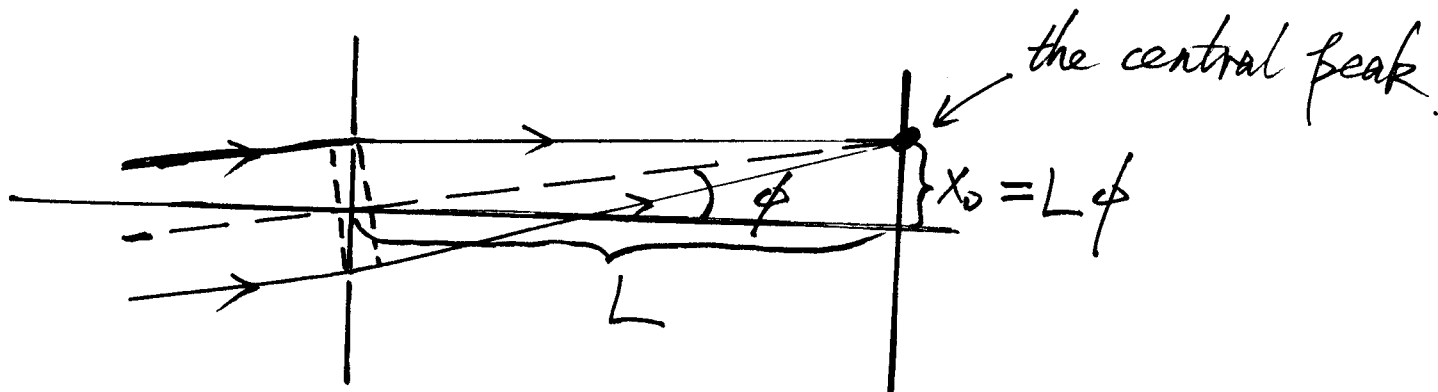
$$\lambda = 1 \text{ cm} = 0.01 \text{ m}$$

$$L = 100 \text{ m}$$

$$\Delta X = 1 \text{ m}$$

$$d = \frac{\lambda L}{\Delta X} = \frac{0.01 \times 100}{1} = 1 \text{ (m)}$$

(b)



the pattern is shifted by $x_0 = L\phi$.

(c)

$$\frac{d}{D} = 3$$

actually $\frac{d}{D} = \frac{3}{n}$

$n = 1$ or 2

but most students assumed $n = 1$, so...

Rubric

(a)

$$\left. \begin{aligned} \Delta\varphi &= \frac{2\pi}{\lambda} d \sin\theta \approx \frac{2\pi}{\lambda} d\theta \\ \Delta\varphi &= m \cdot 2\pi \end{aligned} \right\} \Rightarrow d\theta = m\lambda$$

$$\theta \approx \frac{x}{L} \Rightarrow d \frac{x}{L} = m\lambda$$

$$\Delta x \frac{d}{L} = \Delta m \cdot \lambda = \lambda$$

\Downarrow

$$\Delta x = \lambda \frac{L}{d}$$

Or equivalent deduction process to get this relation : 6 points

$$\Delta x = 2\lambda \frac{L}{d}$$

-3

No deduction but wrote down $d = \frac{\lambda L}{\Delta x}$

and with correct final value of d : 10 points.

Making mistakes when plugging numbers into

correct equation : -2

(b) picture 2.

shifted only 1

the amount of shift $x_0 = L\phi$ 2.

(c) missing order $m=3$ if wrong -4 points

$\frac{d}{D} = m$ if wrong -4 points

these are just in case that the
final answer is incorrect

But if correct final answer $\frac{d}{D} = 3$ is
got, then full points.

Problem 3!

Solution

(a) $d_o \rightarrow \infty$, $d_i = f$.

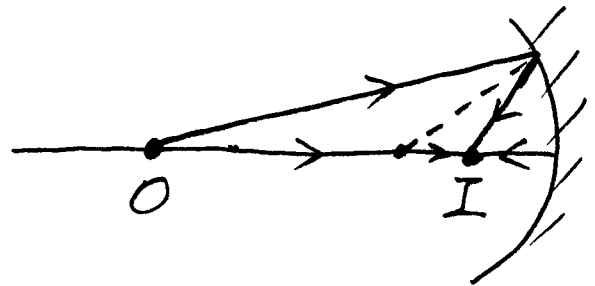
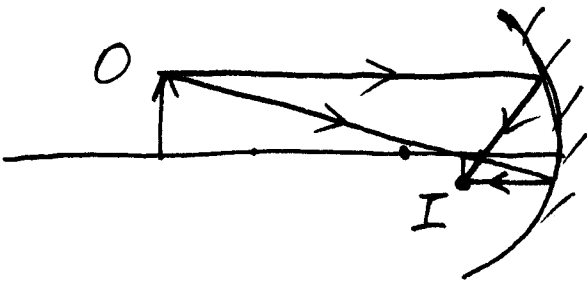
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$f = f$$

$$f = \frac{R}{2}, \quad R = 2f = 2m.$$

(b)

Or



Real Image.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow \frac{1}{5} + \frac{1}{d_i} = \frac{1}{1} \Rightarrow d_i = \frac{5}{4} \text{ m}$$

(c) $\frac{1}{5} + \frac{1}{d_i} = \frac{1}{-1} \Rightarrow d_i = -\frac{5}{6} \text{ m}$

Virtual Image.

Rubric

(a) $f = 1\text{ m}$ 3 points
 $R = 2\text{ m}$ 2 points

if final answer incorrect, then

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad 2 \text{ points}$$

$$f = \frac{R}{2} \quad 1 \text{ point.}$$

(b) picture 5 points

real image 5 points

$$d_i = \frac{5}{4}\text{ m} \quad 5 \text{ points}$$

if value of d_i incorrect, then

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad 3 \text{ points.}$$

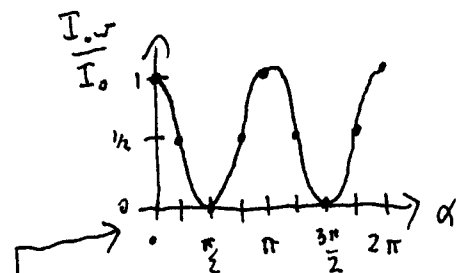
(c) $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ 2 points

$$d_i = -\frac{5}{6}\text{ m} \quad 1 \text{ point} \quad \text{virtual image } 2 \text{ points}$$

Solutions and Rubric for #4:

4(a)

Equation for solution is $I_{out} = I_0 \cos^2 \alpha$, where α is the relative angle between polarization axis and \hat{x} . (+5 pts for equation)



Only graph $[0, 2\pi)$, per directions!

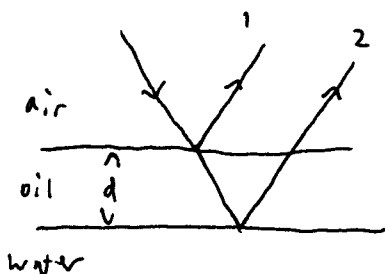
$I_{max} = I_0, \quad I_{min} = 0 \quad (+1 \text{ pt.})$

+4 pts for graph (with correct axes, units).

I also accept plots against absolute angle, so long as you specify initial orientation of pol. axis,

or indication of knowledge thereof, depicted in graph.

4(b)



(angles in figure are simply schematic)

$\phi_2 - \phi_1 = 2\pi m \quad (+1 \text{ for indication of condition for constructive interference, in any valid fashion})$

$\phi_1 = \pi \text{ since } n_o > 1 \quad (+2 \text{ for identification that ray 1 undergoes phase change due to reflection})$

$\phi_2 = \pi + 2d \frac{2\pi n_o}{\lambda} \quad (+2 \text{ for ray 2 phase change upon reflection,}$

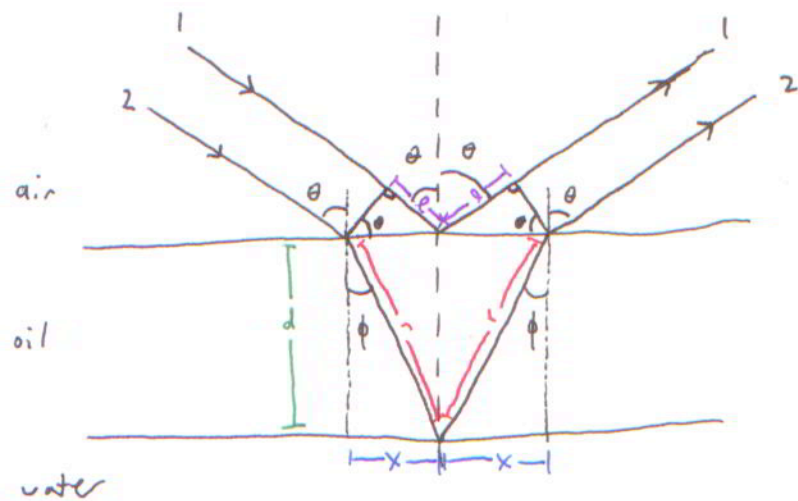
$\rightarrow d = \frac{1}{2} \lambda n_o^{-1} m$

for $m=1, 2, \dots$

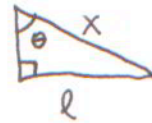
(+1 for correct final result, including correct values for m)

+2 for correct phase due to path traveled. one point taken off for forgetting " n_o "
 ↑ another point off for forgetting factor of 2.

4.c Let's try this again ... 😊



Zoom-in on triangle with "l":



(after rotating)

Snell: $\sin \theta = n_o \sin \phi \rightarrow \sin \phi = \frac{1}{n_o} \sin \theta$

$\phi_1 = 2l \frac{2\pi}{\lambda} + \pi$; $\phi_2 = 2r \frac{2\pi n_o}{\lambda} + \pi$.

Useful trig: $\sin \theta = \frac{l}{x}$, $\cos \phi = \frac{d}{r}$, $\tan \phi = \frac{x}{d}$

$\rightarrow l = x \sin \theta = d \sin \theta \tan \phi = \frac{d}{n_o} \frac{\sin^2 \theta}{\sqrt{1 - \frac{1}{n_o^2} \sin^2 \theta}}$,

where $\cos \phi = \sqrt{1 - \sin^2 \phi} = \sqrt{1 - \frac{1}{n_o^2} \sin^2 \theta}$.

$r = \frac{d}{\sqrt{1 - \frac{1}{n_o^2} \sin^2 \theta}}$

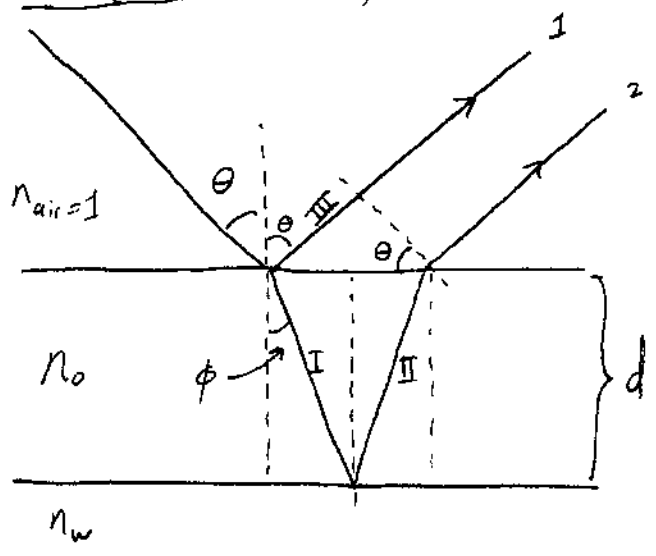
$\phi_2 - \phi_1 = \frac{2\pi}{\lambda} \cdot 2 \cdot \left(n_o \frac{d}{\sqrt{1 - \frac{1}{n_o^2} \sin^2 \theta}} - \frac{1}{n_o} \frac{d \sin^2 \theta}{\sqrt{1 - \frac{1}{n_o^2} \sin^2 \theta}} \right)$

$= \frac{2\pi}{\lambda} 2 n_o d \frac{1}{\sqrt{1 - \frac{1}{n_o^2} \sin^2 \theta}} \left(1 - \frac{1}{n_o^2} \sin^2 \theta \right)$

$\Rightarrow \frac{2\pi}{\lambda} 2 n_o d \sqrt{1 - \frac{1}{n_o^2} \sin^2 \theta} = 2\pi m$

$\hookrightarrow d = \frac{\lambda}{2 n_o} \frac{1}{\sqrt{1 - \frac{1}{n_o^2} \sin^2 \theta}} m$; $m = 1, 2, 3, \dots$

4c) Note: Any other pair of rays can also be chosen!



For example

For constructive interference need the condition:

$$\left(\text{Phase acquired along segments I + II} \right) - \left(\text{Phase acquired along Segment III} \right) = 2\pi m, \quad m \in \mathbb{Z}^+ - \{0\}$$

This piece missing in old solution.

$$\Rightarrow \frac{2\pi n_0 d}{\lambda \cos \phi} - \frac{2\pi}{\lambda} (2d \tan \phi) \sin \theta = 2\pi m$$

use $\sin \theta = n_0 \sin \phi$

$$\Rightarrow \frac{2dn_0}{\cos \phi} (1 - \sin^2 \phi) = 2dn_0 \cos \phi = m\lambda$$

or $d = \frac{m\lambda}{2n_0 \cos \phi}$

$$\boxed{d = \frac{m\lambda}{2\sqrt{n_0^2 - \sin^2 \theta}}}$$

$m \in \{1, 2, 3, \dots\}$