# Solutions.

## problem 1.

(a) $$T = \sqrt{\frac{1+\beta}{1-\beta}}\, T_0 = \sqrt{\frac{c+V}{c-V}}\, T_0 = \sqrt{\frac{c+V}{c-V}} \ (s)$$

Note, this is the inverse of part b. It is not just time dialation (this would be the case if the motion is $\perp$ to the line of sight), because the source moves away as well between the pulses.

(b) $$f = \sqrt{\frac{1-\beta}{1+\beta}}\, f_0 = \sqrt{\frac{c-V}{c+V}}\, f_0 = \sqrt{\frac{c-V}{c+V}}\ 10^6 (Hz)$$

(c) $$L = \frac{1}{\gamma} L_0 = \sqrt{1-\beta^2}\, L_0 = \sqrt{1-(\tfrac{V}{c})^2}\, L_0 = \sqrt{1-(\tfrac{V}{c})^2}\ 10^3 (m)$$

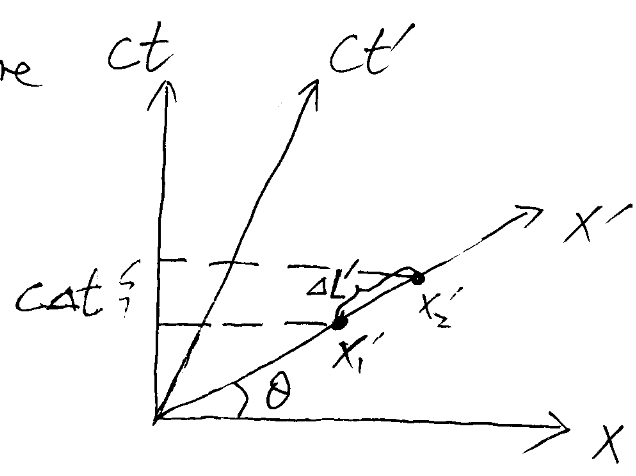
(d) No. the back end one happens first.

$$\Delta t = t_2 - t_1 = \gamma\left[(t_2'-t_1') + \frac{V}{c^2}(x_2'-x_1')\right]$$

$$= \gamma \frac{V}{c^2}(x_2'-x_1') = \gamma \frac{V}{c^2} L_0$$

$$= \frac{1}{\sqrt{1-(\frac{V}{c})^2}} \frac{V}{c^2} L_0 = \frac{1}{\sqrt{c^2-V^2}} \frac{V}{c}\, 1000 (s).$$

picture




$\tan\theta = \beta$

$c\Delta t = \Delta L' \cdot \sin\theta$

$\Delta L' = (x_2'-x_1')|\vec{e}_{x'}|$

$= (x_2'-x_1')\gamma \sec\theta$

$\therefore c\Delta t = (x_2'-x_1')\gamma \tan\theta$

$= \gamma\beta\Delta x' = \gamma\beta L_0$

$\therefore \Delta t = \gamma \frac{\beta}{c} L_0$

# Rubric

problem 1.

(a) time dilation 2 $\gamma(s)$ $-3$
doppler effect 3.

(b) $\sqrt{\frac{c-v}{c+v}}f_0$ instead of $\sqrt{\frac{c-v}{c+v}}\,10^6\,(Hz)$ $-1$

other answers $-5$

(c) with L instead of km $-1$

(d) with L $-1$

incorrect value $-5$

wrong picture $-2$

wrong explanation $-1$

wrong first one $-1$

problem 2

(a) $\omega = c|\vec{k}|$

$\omega = 3\times10^{8}\ s^{-1}$

(b) $\lambda = \frac{2\pi}{|\vec{k}|} = 2\pi\,(m)$

(c) $\vec{E}_0 \perp \vec{k}$

$\vec{B}(\vec{r},t) = \frac{1}{\omega}\vec{k}\times\vec{E}_0\cos(\omega t - \vec{k}\cdot\vec{r} + \phi)$

(d) from $\omega_0$ to $4\omega_0$, increasing by $3\omega_0$

(e) $v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{3\epsilon_0\mu_0}} = \frac{1}{\sqrt{3}}c$

$\omega = v|\vec{k}| = \frac{1}{\sqrt{3}}c|\vec{k}|$

problem 2

(a) $\omega = c|\vec{k}|$ 3

$\omega = 3\times10^{8}\,s^{-1}$ 2

(b) $\lambda = \frac{2\pi}{|\vec{k}|}$ 3

without unit −1

(c) $\vec{E}_0 \perp \vec{k}$ 1

$\vec{B}(\vec{r},t)$ 2

$\vec{B}$ direction 2

(d) 5

(e) $\checkmark$ 3

$\omega = \frac{1}{\sqrt{3}} c|\vec{k}|$ 2 problem 3.

(a) $E_1 = \gamma_1 m_1 c^2 = \frac{5}{4} mc^2$

$E_2 = \gamma_2 m_2 c^2 = \frac{5}{6} mc^2$

(b) $p_1 = \gamma_1 m_1 V_1 = \frac{3}{4} mc$

$p_2 = -\gamma_2 m_2 V_2 = -\frac{2}{3} mc$.

(c). $p = \frac{1}{12} mc$.

$M = \frac{\sqrt{39}}{3} m$

$E_k = \frac{25-4\sqrt{39}}{12} mc^2$.

problem 3.

(a). $\gamma_1 = \frac{5}{4}$ 1

$\gamma_2 = \frac{5}{3}$ 1

$E_1 = \frac{5}{4}mc^2$ $E_1 = \gamma_1 m_1 c^2$, without $\frac{5}{4}mc^2$ −1

$E_2 = \frac{5}{6}mc^2$ $E_2 = \gamma_2 m_2 c^2$. = −1

(b). $p_1 = \gamma_1 m_1 V_1$ 1

$p_2 = -\gamma_2 m_2 V_2$ 2

wrong sign −1

value 1+1

(c) $p = p_1 + p_2$ 3 value($p$) 2

$E = E_1 + E_2$ 2 $M = m_1 + m_2$ −3

$E^2 = p^2c^2 + M^2c^4$ 2 value($M$) 1

$E_k = E - Mc^2$ 3 value($E_k$) 2

Problem 4

a) Conservation of Energy: $E_{TOT\ before} = E_{TOT\ after}$

$$M_{before}c^2 + \underbrace{KE_{before}}_{0} = M_{after}c^2 + KE_{after}$$

$$\boxed{KE_{after} = (M_{befor} - M_{after})c^2 = 18 \text{ MeV}}$$

---

b) To see if non-relativistic, want to approxamate:

$\left\{ E_{TOT} \approx mc^2 + \frac{1}{2}\frac{p^2}{m} \right.$ ← per particle

$$E_{TOT} = \sqrt{p^2c^2 + m^2c^4} = mc^2\sqrt{1 + \frac{p^2}{m^2c^2}} \approx mc^2\left(1 + \frac{1}{2}\frac{p^2}{m^2c^4}\right)$$

is satisfied ⟹ for $\frac{p^2}{m^2c^2} << 1$ or $\frac{2 \times \text{Non Rel KE}}{mc^2} << 1$

For estimate of this since, both particles same order of mass can take $KE \sim \frac{18}{2}$ MeV, so our condition becomes:

$\frac{.018\,\text{GeV}}{1\,\text{GeV}} << 1$ which is true!

Also can solve problem assuming Non Rel and confirm the assumption in retrospect:

Solution: call $m \equiv m_n$ = mass of neutron

$$\begin{cases} mV_n = 4mV_{He} & \Longrightarrow V_{He} = \frac{V_n}{4} \\ \frac{1}{2}mV_n^2 + 2mV_{He}^2 = 18 \text{ MeV} \end{cases}$$

$$\Longrightarrow \frac{1}{2}V_n^2 + \frac{1}{8}V_n^2 = \frac{18 \text{ MeV}}{m} = .018\,c^2$$

$$\Longrightarrow \boxed{\begin{array}{l} V_n = \sqrt{\frac{8}{5}(.018)}\,c \approx \sqrt{.029}\,c \approx .17c \\ V_{He} = \frac{1}{4}V_n \approx .043c \quad << c \end{array}}$$

$<< c$ (enough for 5% accuracy)

$$KE_{He} = \frac{1}{4} \times 4 \times \frac{1}{16}\left(\frac{8}{5} \times .018\right) \text{ GeV}$$

$$= \frac{.018}{5} \text{ GeV} = \boxed{.0036 \text{ GeV}}$$

$$\boxed{KE_n = \frac{1}{2} \times \frac{8}{5} \times .018 \text{ GeV} = .0144 \text{ GeV}}$$

---

c) Now $E_{TOT} = \sqrt{p^2c^2 + m^2c^4} \approx pc + \mathcal{O}\left(\frac{mc^2}{pc}\right)$

estimate $\frac{mc^2}{pc} = \frac{1}{\gamma\beta}$ but for neutrino $m_n c^2(\gamma_n - 1) \approx \frac{10^3}{2} m_n c^2$

$\gamma_n \sim \frac{10^3}{2}$ $\beta \sim 1$

$$\therefore \frac{1}{\gamma\beta} = \frac{2}{10^3} << 1$$

$\therefore$ They are ultra relativistic or approxantly massless!

Each move w/ equal $E_{TOT} \approx E_{KE} = pc$

$\gamma_n = \gamma_{He} = \frac{10^3}{2}$ $\boxed{KE_n = KE_{HE} = \frac{10^3}{2} \text{ GeV} = 500 \text{ GeV}}$

$$\beta_n = \beta_{He} = \sqrt{1 - \frac{1}{\gamma^2}} \approx 1 - \frac{1}{2}\frac{1}{\gamma_n^2}$$

$$= \left(1 - 2 \times 10^{-6}\right)$$

$$\boxed{V_n \approx V_{He} \approx \left(1 - 2 \times 10^{-6}\right)c}$$

# Problem 4 Rubric

a) 5 pts

(-1 to -3) for no justification

writing $\begin{cases} E = mc^2 \\ E_{KE} = 18 \text{ MeV} \end{cases}$ or similar (-3 pts)

(10 pts)

b) No justification for approx (all else correct) (-2 pts)

Got V or KE correctly but forgot other (-2)

(10 pts)

c) Correct conservation laws (+3)

Got V or KE but forgot other (-3)

No approximation and couldn't do algebra (-4)