Mueller MT2 Problem 1

a. In one hour, a mass of

$$(1.1\times10^{6})(1.1\times10^{3}\ kg/m^{3}) = 1.1\times10^{9}\ kg$$

of water falls. This generates

$$(.85)\left((1.1\times10^{9}\ kg)(9.8\ m/s^{2})(350\ m)\right) = 3.20\times10^{12}\ \text{Joules}$$

of energy (taking efficiency into account). — 3 points

Finally, the power is

$$\frac{\Delta E}{\Delta t} = \frac{3.20\times10^{12}\ J}{3600\ s} = 8.9\times10^{8}\ W.$$ 2 points

b. It takes

$$(1.1\times10^{9}\ kg)(9.8\ m/s^{2})(350\ m) = 3.77\times10^{12}\ J$$

of energy to pump the water up. The plant consumes 150×10^{6} W of power, but only uses

$$.85(150\times10^{6}\ W) = 130\ W$$

So it takes

$$\Delta t = \frac{\Delta E}{P} = \frac{3.77\times10^{12}\ J}{130\ W} = 2.9\times10^{4}\ s = 8.24\ \text{hours}$$

c. Efficiency is given as

$$\eta = \frac{(\text{Energy out})}{(\text{Energy in})} = \frac{(.85)(.85)(\text{Energy in})}{(\text{Energy in})} \cong .72$$

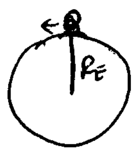
Alternatively, the energy put in is

$$(150\times10^{6}\ W)(2.9\times10^{4}\ s) = 4.4\times10^{12}\ J$$

The energy out, from part (a) is 3.20×10^{12} J, so

$$\eta = \frac{3.2\times10^{12}\ J}{4.4\times10^{12}\ J} \approx .72$$ 5 pts

oblem 2 Mueller Midterm 2 Spring 2010

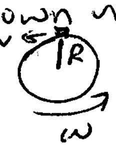
a)  $R_E = 7\times10^3$ km, $g = 9.8$ m/s²

For a satellite orbiting just above ground,

$\Sigma F_c = ma_c = \frac{mv^2}{R} = mg$. (Gravitational acceleration is the centripetal acceleration, and near earth's surface $a_{grav} = g$.)

$\Rightarrow v = \sqrt{Rg} = \sqrt{(7\times10^6\,\text{m})(9.8\,\text{m/s}^2)} \approx \underline{8282.5\ \text{m/s}}$.

The kinetic energy of the satellite is $\frac{1}{2}mv^2 = \frac{1}{2}mgR = \underline{m\,3.43\times10^7\ \text{J}}$.

b)  ; top down view 

The velocity of the rocket relative to the center of the earth when it is just sitting on the surface is found with $v' = R\omega$, where ω is the angular velocity of earth. We can obtain ω by using the period of earth's rotation, which is one day:

$\omega = 2\pi/T = 2\pi/24\cdot60\cdot60 = 7.27\times10^{-5}\,\text{s}^{-1}$.

Then the velocity (magnitude) of the rocket relative to the center of the earth, when the rocket is sitting at the equator, is $v' = R\omega = (7\times10^6\,\text{m})(7.27\times10^{-5}\,\text{s}^{-1}) = \underline{509\ \text{m/s}}$, which is the amount

the necessary velocity is reduced (for going into orbit).

Relative to the launch pad, the remaining velocity needed to enter an orbit close to earth's surface is $\Delta v = |v' - v| = |R\omega - \sqrt{Rg}| = \underline{7774\ \text{m/s}}$

The kinetic energy of the satellite relative to the launch pad is

$\frac{1}{2}m(R\omega - \sqrt{Rg})^2 = \frac{1}{2}m(R^2\omega^2 - 2R\omega\sqrt{Rg} + Rg) = \underline{3.022\times10^7\,m\cdot\text{J}}$

Then the necessary kinetic energy for orbit is reduced by

(original) − (new) $= \cancel{\frac{1}{2}mRg} - \frac{1}{2}mR^2\omega^2 + mR\omega\sqrt{Rg} - \cancel{\frac{1}{2}mRg} = \underline{m\cdot4.09\times10^6\ \text{J}}$

Mueller Problem 2

As a percentage of the energy required in part a, the kinetic energy is reduced by $\Delta KE / KE_{org} \times 100 = 100 \times \frac{4.09\times10^6}{3.43\times10^7} = 12\%$.

c) $g_m = 1/6\, g$; $\rho_m = \rho_E$. We want r_m.

$$\rho_m = \rho_E \Rightarrow \frac{M_m}{\cancel{4/3\,\pi} R_m^3} = \frac{M_E}{\cancel{4/3\,\pi} R_E^3} \Rightarrow M_E = M_m \frac{R_E^3}{R_m^3}$$

The force acting on some mass m at the surface of each is

random little mass; random little mass; moon (subscript)

$$① \ \frac{G m M_E}{R_E^2} = m g_E, \quad ② \ \frac{G m M_m}{R_m} = m g_m$$

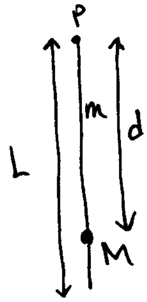
Divide

$$\frac{①}{②} = \frac{\frac{\cancel{G} M_E \cancel{m}}{R_E^2}}{\frac{\cancel{G} M_m \cancel{m}}{R_m^2}} = \frac{\cancel{m} g_E}{\cancel{m} g_m} = 6$$

$$\Rightarrow \frac{\cancel{M_m} R_E^{\cancel{3}} / R_m^{\cancel{3}}}{\cancel{R_E^2} \cancel{M_m} / \cancel{R_m^2}} \Rightarrow R_E = 6 R_m. \Rightarrow \underline{R_m = 1.167\times10^6\ m}$$

Lecture 3

3.) a.)



$L = 1.100$ m

$d = 1.021$ m

$M = 5.000$ kg

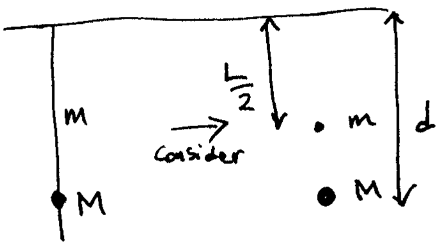
$m = 1.000$ kg

$$I_p = \underbrace{\frac{1}{12} m L^2 + m \left(\frac{L}{2}\right)^2}_{+2} + \underbrace{M d^2}_{+2}$$

$$I_p = \frac{1}{3} m L^2 + M d^2$$

$$I_p = \underbrace{5.616 \text{ kgm}^2}_{+1}$$

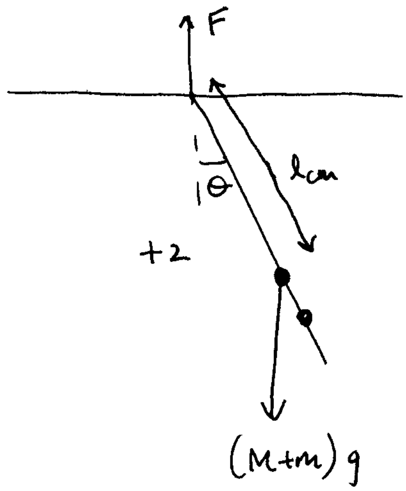
b.)



$$l_{cm} = \underbrace{\frac{m \frac{L}{2} + M d}{m + M}}_{+4}$$

$$l_{cm} = \underbrace{0.9425 \text{ m}}_{+1}$$

c.)



+2

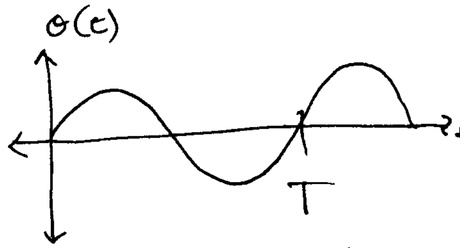
$$\tau = -(M+m) g \, l_{cm} \sin\theta$$

$$\tau \approx \underbrace{-\left(m \frac{L}{2} + M d\right) g \, \theta}_{+3} \approx 5.655 \, g \, \theta \approx 55.4 \theta$$

$$\tau = I \alpha = I \frac{d^2\theta}{dt^2}$$

d.)

$$\underbrace{\frac{d^2\theta}{dt^2} = \frac{\tau}{I}}_{+1} = -\frac{\left(m \frac{L}{2} + M d\right) g}{\frac{1}{3} m L^2 + M d^2} \theta$$



T is the period

T = ~~[illegible]~~ 2.000 sec

Take $\theta(t) = A \sin\left(\frac{2\pi t}{T}\right)$

$$\frac{d\theta}{dt} = \frac{2\pi}{T} A \cos\left(\frac{2\pi t}{T}\right)$$

$$\frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 A \sin\left(\frac{2\pi t}{T}\right)$$

$$\underbrace{\frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \theta}_{+2} \rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{m \frac{L}{2} + M d}{\frac{1}{3} m L^2 + M d^2} g \rightarrow \underbrace{T = 2\pi \sqrt{\frac{\frac{1}{3} m L^2 + M d^2}{\left(m \frac{L}{2} + M d\right) g}}}_{+2}$$

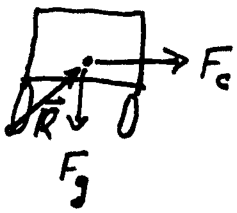
4 Mueller

a) Since power is constant

$\frac{1}{2}mv^2 = KE = P \cdot t$

$$t = \frac{\frac{1}{2}mv^2}{P} = 3.7s$$

b) Looking at the front of the SUV:



Both the centrifugal force and gravity can be taken at the C.o.M.

$$F_c = \frac{mv^2}{r}$$

$$|\tau_c| = |\vec{R} \times \vec{F_c}| = \frac{mv^2}{r} h_{cm} = \frac{2.0 \times 10^6 \ kg\,m^3/s^2}{r}$$

c) It begins to tip over when

$$mg \cdot \frac{w}{2} = |\tau_g| = |\tau_c| = \frac{mv^2}{r} h_{cm}$$

$$r = \frac{2v^2 h_{cm}}{gw} = 68m$$

5

Mueller
MT2

a) $\boxed{V_{CM,x} = \frac{M V_M}{M+m} = \frac{40}{3} \text{ mph}}$

$\boxed{V_{CM,y} = \frac{m V_m}{M+m} = \frac{20}{3} \text{ mph}}$

No external forces $\Rightarrow \vec{V}_{CM}$ same before and after collision

b) Before collision:

- Rest frame: $KE_{tot} = \frac{1}{2} M V_M^2 + \frac{1}{2} m V_m^2$

$$= \frac{1}{2}(2400 \text{ kg})\left(\frac{80}{9} \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(1200 \text{ kg})\left(\frac{80}{9}\frac{\text{m}}{\text{s}}\right)^2$$

$$= \boxed{142222 \text{ J}}$$

- CM frame: $KE_{tot} = \frac{1}{2} M\left[\left(20 - \frac{40}{3} \text{ mph}\right)^2 + \left(\frac{20}{3} \text{ mph}\right)^2\right]$

$$+ \frac{1}{2} m\left[\left(\frac{40}{3} \text{ mph}\right)^2 + \left(20 - \frac{20}{3}\right)^2\right]$$

$$= \boxed{63210 \text{ J}}$$

After collision:

- Rest frame: $KE_{tot} = \frac{1}{2} M V_{Mf}^2 + \frac{1}{2} m V_{mf}^2$

$$= \frac{1}{2} M\left[\left(\frac{40}{3} \text{ mph}\right)^2 + \left(\frac{20}{3} \text{ mph}\right)^2\right]$$

$$+ \frac{1}{2} m\left[\left(\frac{40}{3} \text{ mph}\right)^2 + \left(\frac{20}{3} \text{ mph}\right)^2\right]$$

$$= \boxed{79012 \text{ J}}$$

- CM frame:

Both cars move with CM frame after collision so

$$\boxed{KE_{tot} = 0}$$

c) $\boxed{\vec{V}_M = \vec{V}_m = \left(\frac{40}{3} \text{ mph}, \frac{20}{3} \text{ mph}\right)}$

[redacted]

5 cont.

Mueller
MT2

d) $$a_{M,x} = \frac{\Delta V_{Mx}}{50\ ms} = \frac{\left(\frac{40}{3} - 20\right)mph}{50\ ms} = \frac{\left(-\frac{20}{3}\ mph\right)}{50\ ms}$$

$$= -59.26\ \frac{m}{s^2}$$

$$a_{M,y} = \frac{\Delta V_{My}}{50\ ms} = \frac{\frac{20}{3}\ mph}{50\ ms} = 59.26\ \frac{m}{s^2}$$

$$\Longrightarrow \quad a_M = \sqrt{(-59.26)^2 + (59.26)^2} = \boxed{83.8\ \frac{m}{s^2}}$$

$$a_{m,x} = \frac{\Delta V_{m,x}}{50\ ms} = \frac{40/3\ mph}{50\ ms} = 118.52\ \frac{m}{s^2}$$

$$a_{m,y} = \frac{\Delta V_{m,y}}{50\ ms} = \frac{\left(\frac{20}{3} - 20\right)mph}{50\ ms} = \frac{-40/3\ mph}{50\ ms}$$

$$= -118.52\ \frac{m}{s^2}$$

$$a_m = \sqrt{(118.52)^2 + (-118.52)^2} = \boxed{167.6\ \frac{m}{s^2}}$$

Note: $F_{M\to m} = F_{m\to M}$ so $M a_M = m a_m$

$$\Longrightarrow \quad \frac{a_m}{a_M} = \frac{M}{m} = 2$$

6. a. The power P is converted into kinetic energy $K(t)$, so $P = \frac{dK}{dt}$

then $K = \int dK = \int P\,dt = P \cdot t$ (since P is const.)

$$K(t) = \frac{1}{2} m (v(t))^2 = P \cdot t$$

$\Rightarrow v(t) = \sqrt{\frac{2Pt}{m}}$

Note: $a = \frac{dv}{dt}$ is <u>not</u> constant!
This means you cannot use $x = \frac{1}{2}at^2$!

Displacement $x(t) = \int_0^t v(t')\,dt' = \sqrt{\frac{2P}{m}} \int_0^t t'^{1/2}\,dt' = \frac{2}{3}\sqrt{\frac{2P}{m}}\, t^{3/2}$

Let D be distance between stop signs and T be the time to travel between them. Then:

$x(T) = D = \sqrt{\frac{8P}{9m}}\, T^{3/2}$ → solve for T

$T = D^{2/3} \left(\frac{9m}{8P}\right)^{1/3}$ for each stop sign

$$1 \text{ mile} = (6 \times \underbrace{250 \text{ meters}}_{D}) + \underbrace{109.3}_{D_{end}} \text{ meters}$$

$$T_{tot} = 6 \times T + T_{end} = \left(\frac{9m}{8P}\right)^{1/3} \left(6D^{2/3} + D_{end}^{2/3}\right) = 193 \text{ sec.}$$

$= 3.2$ min.

6. b. Constant v means that your power output is just enough to cancel frictional losses:

$$dW = \vec{f} \cdot d\vec{x} = -f\,dx$$

$$P = -\frac{dW}{dt} = +f\frac{dx}{dt} = f\,v$$

velocity $V = \frac{1\,mi}{T_{tot}} = \frac{1609.3}{193}\,\frac{m}{s} = 8.34\ m/s$

$= 18.7\ mi/hr$

It is not strictly correct to describe the friction by a single coefficient. There is energy loss due to the kinetic friction at each moving part of the bicycle, as well as deformation of the wheels and air resistance. However, we will approximate the friction, using the effective coefficient μ, as $f = mg\mu$.

Then: $P = mg\mu v$

$$= 90\,kg * 9.8\,\frac{m}{s^2} * 0.05 * 8.34\ m/s$$

$$= 368\ W$$