IEOR 161 Operations Research II
University of California, Berkeley
Spring 2007

Midterm 2 Solution

1. (a) Let the states be the person Paris has chosen to go out with, i.e. $\{N, B, L, S\}$. Therefore the transition probabilities are

$$\begin{pmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 1/3 & 0 & 1/3 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 \end{pmatrix}$$

(b) The Transition Matrix is doubly stochastic, limiting probabilities are $\pi_N = \pi_B = \pi_L = \pi_S = 1/4$. Therefore the proportion of time Paris spends with each friend is $1/4$.

(c) Let T_i, j denote the number of transitions needed to go from i to j.

$$E[T_{N,S}] = 1 + P_{N,B}E[T_{B,S}] + P_{N,L}E[T_{L,S}] = 1 + 1/3(E[T_{B,S}] + E[T_{L,S}])$$

By symmetry, $E[T_{N,S}] = E[T_{B,S}] = E[T_{L,S}]$, thus $E[T_{N,S}] = 3$.

2. (a) Let G be the process of green men's arrival process, and R be the process of red men's arrival process. min(G,R) is exponential with rate μ+λ=2+3=5. Let T_i be the interarrival time between the (i-1)th arrival and ith arrival, and T be the time it takes to fill the space ship.

$$E[T] = \sum_{i=1}^{3} E[T_i] = 3 \times \frac{1}{5} = \frac{3}{5}$$

(b) For each arrival, the probability that it is a green man is

$$Pr\{G < B\} = \frac{\mu}{\mu + \lambda} = \frac{2}{5}$$

Since the arrivals are independent,

$$Pr\{\text{the crew are all green}\} = (\frac{2}{5})^3$$

3. If we count a organ if it is launched before time s but remains alive at time t, then the number of items counted is Poisson with mean $m(s) = \lambda \int_0^s e^{-\mu(t-y)} dy$. The desired probability is $e^{-m(s)}$.