UNIVERSITY OF CALIFORNIA, BERKELEY

DEPARTMENT OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCES

EECS 130 Professor Chenming Hu

Fall 2009

Midterm II Solutions

Name: ______________________

SID:________________________

Closed book. Two sheets of notes are allowed.

*There are **10** pages of this exam including this page.*

Problem 1	25
Problem 2	20
Problem 3	30
Problem 4	25
Total	100

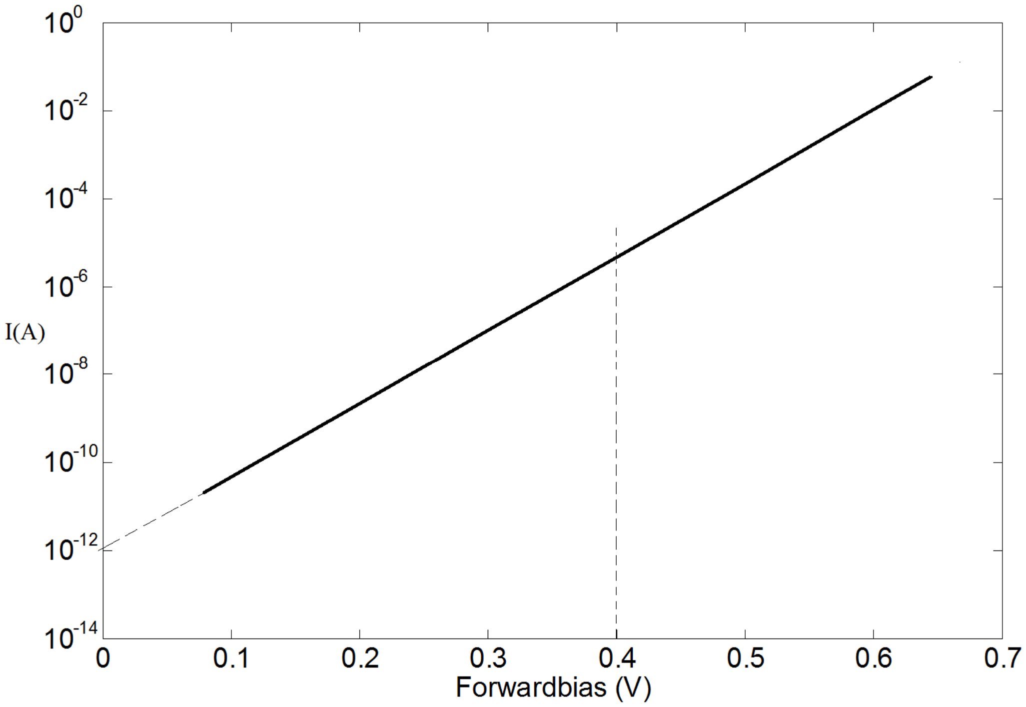
Physical Constants

Electronic charge	q	1.602 x 10^{-19} C
Permittivity of vacuum	ε_0	8.845 x 10^{-14} $F \cdot cm^{-1}$
Relative permittivity of silicon	$\varepsilon_s / \varepsilon_0$	11.8
Relative permittivity of SiO_2	$\varepsilon_{ox} / \varepsilon_0$	3.9
Boltzmann's constant	k	8.617 x 10^{-5} $eV \cdot K^{-1}$ or 1.38 x 10^{-23} $J \cdot K^{-1}$
Thermal voltage at T = 300K	kT/q	0.026V
Effective density of states	N_{c_Si}	2.8 x 10^{19} cm^{-3}
Effective density of states	N_{v_Sli}	1.04 x 10^{19} cm^{-3}
Silicon Band Gap	E_{g_Si}	1.12eV
Intrinsic carrier concentration of Si at 300K	n_{i_Si}	1.5 x 10^{10} cm^{-3}
GaAs Band Gap	E_{g_GaAs}	1.42eV

(Assume T=300K unless otherwise mentioned)

1. **Small Signal Model of P$^+$N Diode**

(a) **(3Pts)** Find the I_0 (reverse saturation current) of the diode from the figure below.



Since PN junction I-V relation $I = I_0(\exp(V/(kT/q))-1)$ *is exponential when V is larger than a few kT/q, the curve is linear with a slope of 60mV/dec in semilogy plot. Extrapolating the I-V curve to intersect the y-axis* ($V_{forward}=0$) *you will get* $I_0 = 1pA$.

(Some students mistakenly treat logI(A) as the value of y axis coordinate, and double count log. Since the plot in the exam is not completely clear, full credits are given as long as we see extrapolation, 1e-12A or a reasonable way to get I_0 in your answer.)

(b) **(3Pts)** At a forward bias of 0.4*V*, what is the small signal conductance of the P$^+$N junction?

$I\ (V_{forward} = 0.4V) = I_0.[\exp(q.V_{forward}/kT)-1] = 4.802\mu A.$

$G = dI/dV = q.I/(kT) = 4.802\mu A\ /0.026V = 1.85x10^{-4}\ \Omega^{-1}$

(I@Vforward=0.4V can also be found in the plot in (a), which is around 5µA. Some students get the wrong numerical answer because of the wrong value of I_0 in (a). We give full credits if we see G = *qI/(kT)*.)

(c) **(3Pts)** At the bias in (b), to achieve a small signal capacitance of 5*nF*, what should the charge-storage time (τ_S) of the diode be?

Diffusion Capacitance, $C = dQ/dV = \tau_S.dI/dV = \tau_S.G$

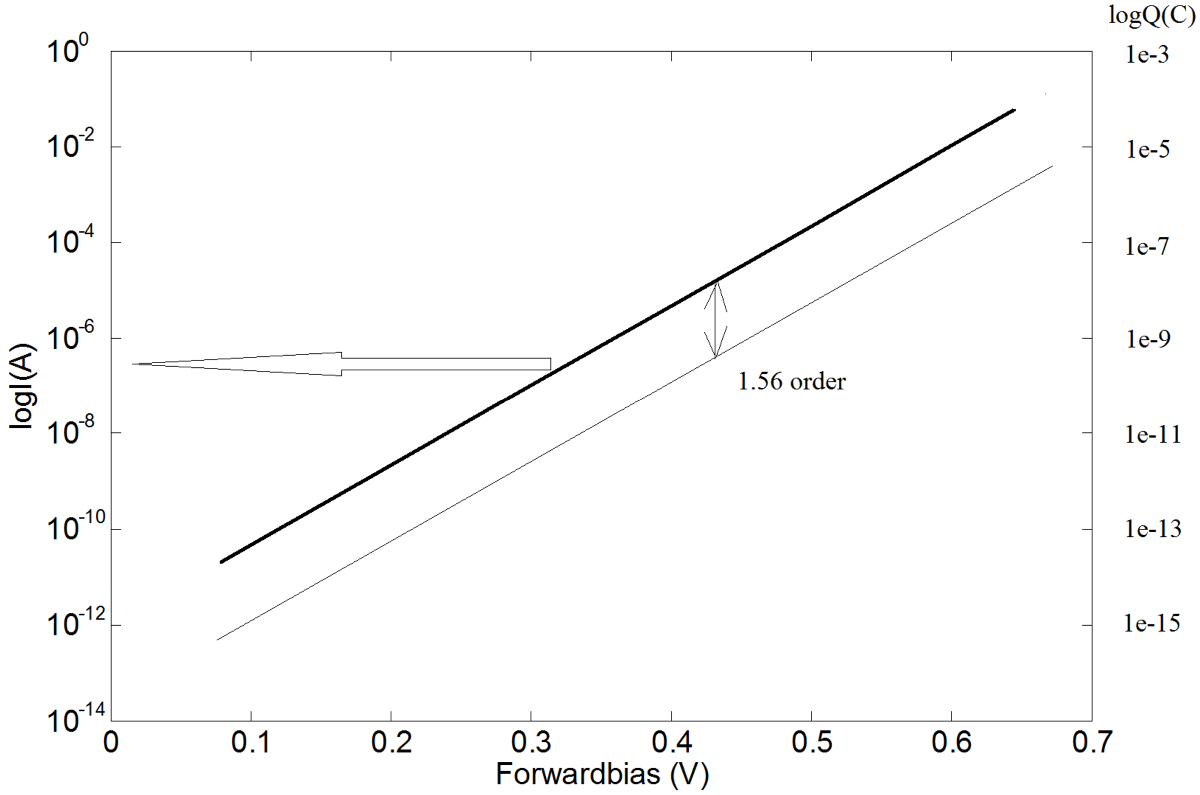
=> *charge-storage time,* $\tau_S = C / G = 27.07\ \mu s$

(Again, since (b) and (c) are coupled, full credits are given if we see $\tau_S = C / G$.)

(d) **(3Pts)** Draw the stored charge Q_S vs. Bias ($V_{forward}$) in the figure below.

$Q_S = I.\ \tau_S$

- ⇨ $log(Q_S) = log(I) + log(\tau_S) \sim log(I) + 10^{-4.567}$
- ⇨ *a line parallel to I …shifted by 4.567 orders below, since y axis on the right is already 3 orders lower than the left y axis, only need to shift down by about 1.567 orders, represented below.*



(parallel: 1pt, shift down: 1pt, order between 1 to 2: 1pt)

(e) **(4Pts)** Suppose the doping, N_d=1e17cm^{-3}, estimate the depletion layer thickness under 0.4*V* of forward bias.

Built in potential, $\varphi_{bi} = Eg/2 + (kT/q).ln(N_d/n_i) = 0.56 + 0.419 = 0.979\ V.$

$$\text{Depletion thickness, } W_{dep} = \sqrt{\frac{2.\varepsilon_0.\varepsilon_{Si}.(\varphi_{bi} - V_A)}{q.N_d}} = 86.53nm$$

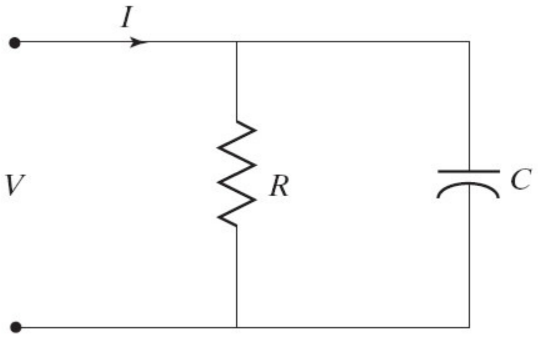
(Those of you who end up with φ_{bi}=0.419V should pay attention to the way to calculate φ_{bi}, especially when one side is heavily doped. Also you should know when it's forward biased, the potential drop across the junction should be $\varphi_{bi} - V_A$.)

(f) **(3Pts)** What is the depletion capacitance at this bias? Is it smaller or larger compared to the diffusion capacitance given in (c)? Assume diode cross-section area = 0.01 cm^2.

$C_{dep} = A.\ \varepsilon_0.\varepsilon_{Si}/W_{dep} = 1.197nF <$ *Diffusion Capacitance, C.*

(In reality, Diffusion Capacitance should be much larger than C_{dep} when moderately forward biased. Again, since (e) and (f) are coupled, we give full credits if we see the right equation.)

(g) **(3Pts)** Draw the RC equivalent circuit of the diode.



(h) **(3Pts)** Calculate the RC time constant of the diode.

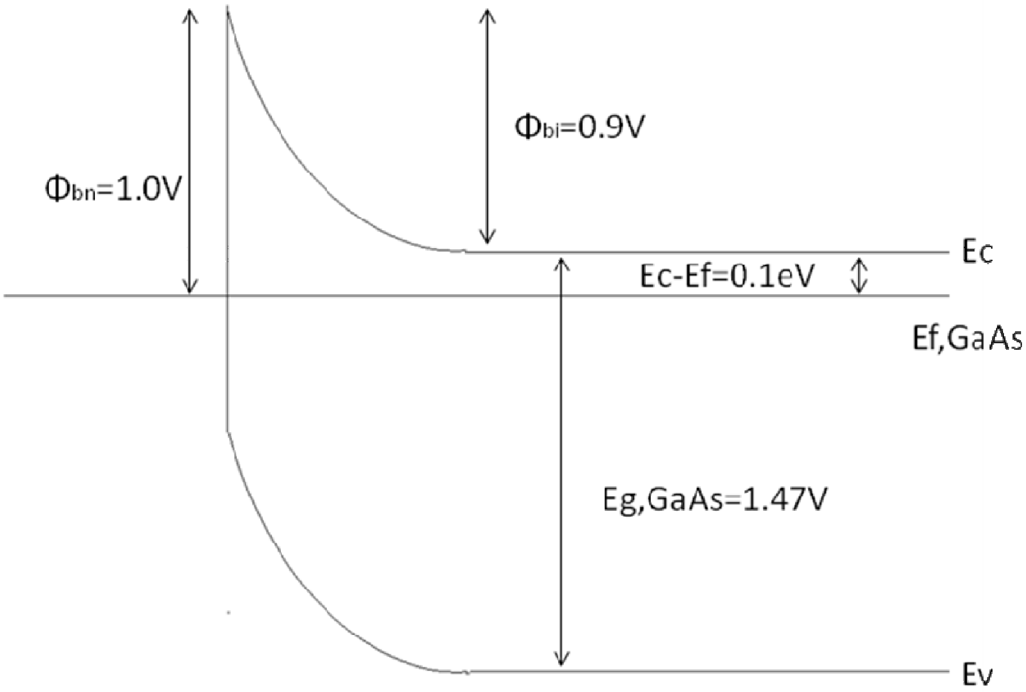
$\tau = R.C = C/G = \tau_S = 27.07\ \mu s$

(τ has to be self-consistent with the τ_S in (c) or slightly larger because of adding C_{dep} in parallel with $C_{diffusion}$)

2. **GaAs Schottky diode and MESFET**

(a) **(5Pts)** GaAs has E_g=1.47eV, N_c=4.7x10^{17}cm^{-3}. Given φ_{Bn}=1.0*V*, doping concentration N_d=10^{16}cm^{-3}, draw the energy diagram for this diode at zero bias condition. Label (Fermi level) E_F and (Built-in potential) φ_{bi}.

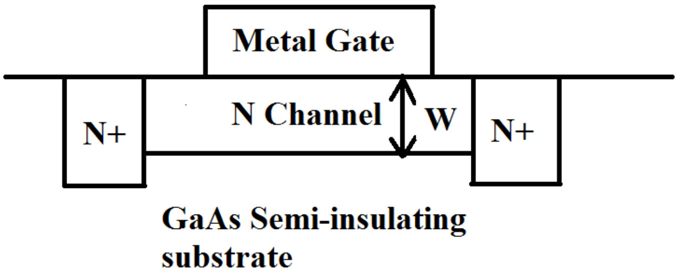
$\varphi_{bi} = \varphi_{Bn} - (kT/q).ln(N_c/N_d) = 1.0 - 0.1 = 0.9\ V$



(b) **(2Pts)** What is the depletion layer thickness, W_{dep}? ($\varepsilon_{GaAs} = 13$)

$$\text{Depletion thickness, } W_{dep} = \sqrt{\frac{2.\varepsilon_0.\varepsilon_{GaAs}.(\varphi_{bi})}{q.N_d}} = 0.36\mu m$$

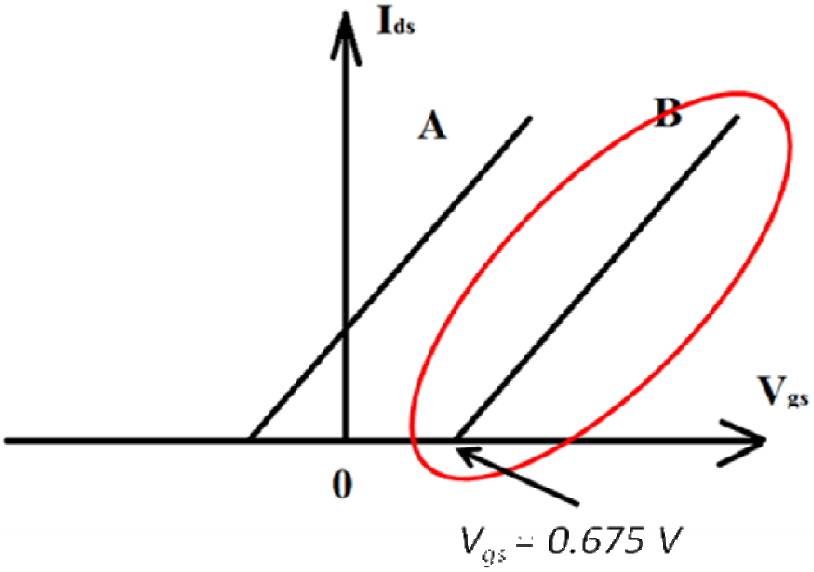
(c) **(5Pts)** Shown below is the structure of a MESFET. What is the maximum value of channel thickness, 'W' that will result in an enhancement-mode transistor?



Enhancement mode => Channel is 'off' at Vg=0V => N Channel is atleast fully-depleted for Vg=0V => maximum W = W_{dep}*. For any value greater than* W_{dep} *the channel would be partially depleted thus allowing current to conduct at Vg=0V.*
So maximum W = $W_{dep} = 0.36\mu m$

(d) **(4Pts)** Which I-V curve below do you think is the right characteristic for a MESFET that has one-half the channel width of that in (c)? Explain in one statement.

Half the channel width => it is indeed enhancement mode … and would switch on only for some positive applied gate bias Vg …when the depletion width reduced to less than half => curve B



(e) **(4Pts)** Find the value of V_{gs} where this transistor (in (d)) turns from off to on and show it on the plot too.

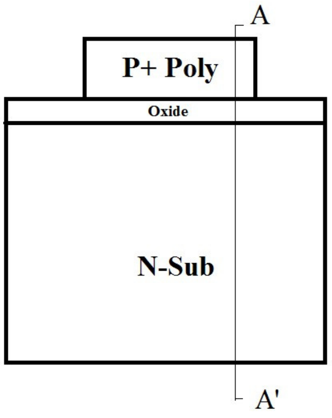
$W_{channel} = W_{dep}/2 = 0.18\mu m$

So transistor turns on for, $W_{dep}(V_{gs}) = W_{dep}(V_{gs}=0)/2 = \sqrt{\dfrac{2.\varepsilon_0.\varepsilon_{GaAs}.(\varphi_{bi} - V_{gs})}{q.N_d}}$

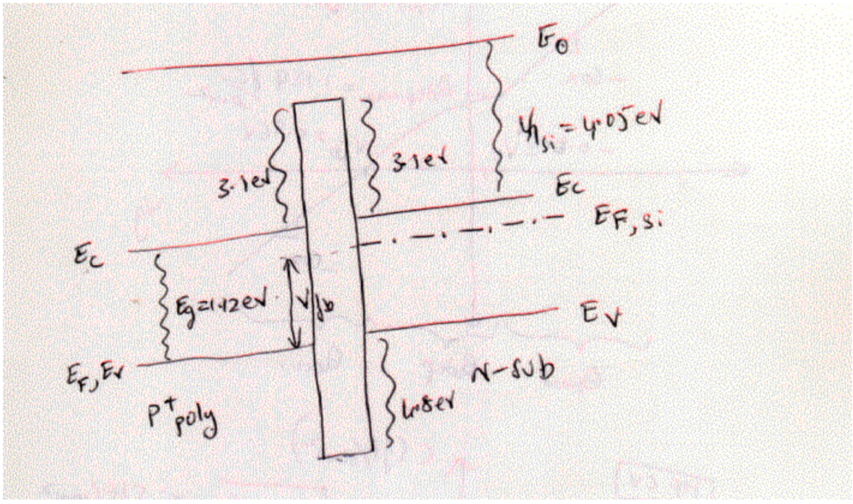
$\Rightarrow V_{gs} = 0.675V$

(Again, forward bias means a minus in the formula.)

3. P+ Poly/ N-substrate MOSCAP



(a) **(5Pts)** Sketch the energy band diagram along the line A-A’ at flat-band voltage.



Sorry for the tilted figure :D
Not looking for Eo and band-offset values
Showing E_C,E_V essentially flat and a larger band-gap oxide – ***2Pts***
*Labels E_C,E_V,$E_{F,Si}$, $E_{F,Poly}$ ---**2Pts***
*Indicating V_{fb} ---**1Pt***

(b) **(4Pts)** Calculate the threshold voltage, given that V_{fb}=0.96*V*, C_{OX}=2fF/μm^2, 2φ_B= 0.8V, Q_{dep_max}=$qN_{sub}W_{dmax}$=1 fC/μm^2. (Ignore Poly-Depletion)

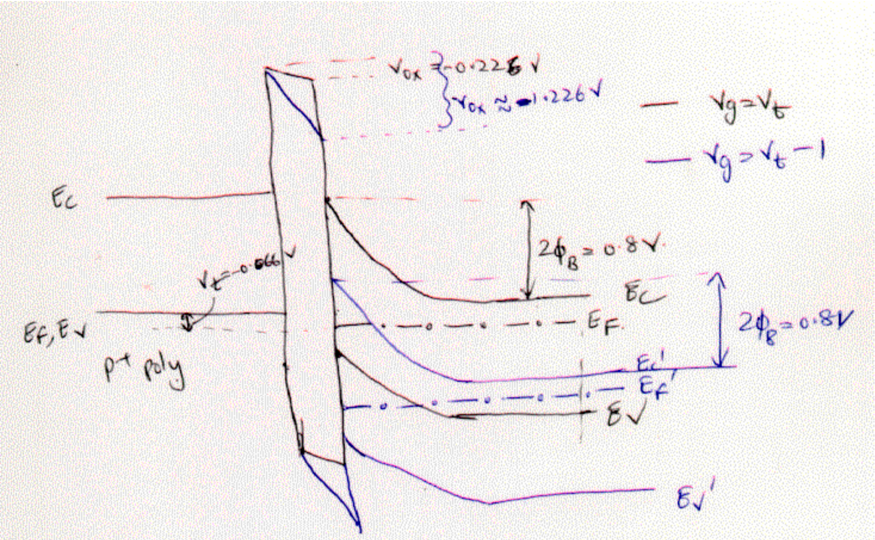
$V_t = V_{fb} - 2\varphi_B - Q_{dep\_max}/C_{ox} = 0.96 - 0.8 – 0.5 = -0.34V$
(It’s OK if you used different values)

Formula ---**2Pts**
Consistent Answer ---**2Pts**

Plots for questions further should be consistent with the answer you have got here.

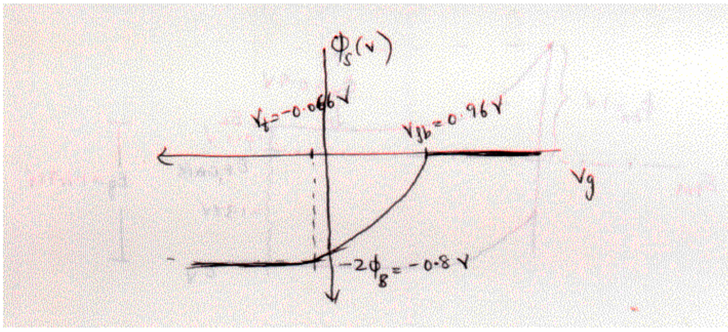
(c) **(5Pts)** Draw energy band diagram at $V_g=V_t$ and $V_g=V_t-1V$, show both diagrams on the same sketch and clearly show their differences.

At $V_g = V_t$ … band-bending in substrate = $2\varphi_B = 0.8V$; and Vox = -0.5V
At $V_g = V_t - 1V$ …. MOSCAP in inversion … band-bending in substrate stays the same $=2\varphi_B = 0.8V$; all the voltage drops appears in the oxide, Vox = -1.5V



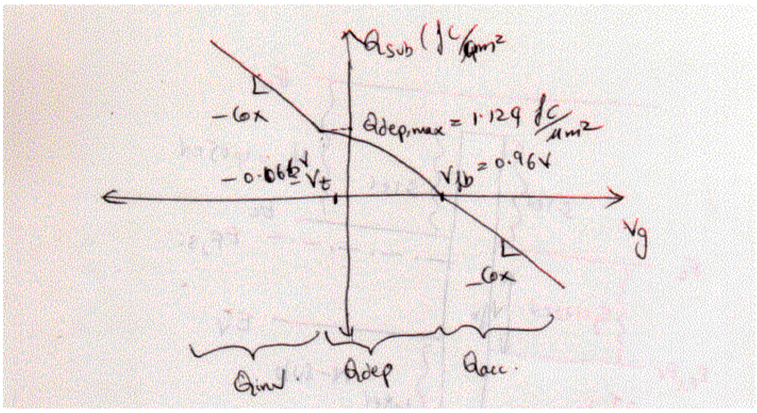
In above figure replace $V_{ox} = -0.5V$ and $V_{ox} = -1.5V$ and $V_t = -0.34V$
*$V_g=V_t$ figure … showing band-bending correctly –**2Pts***
*Labels ---**1 Pt***
*$V_g = V_t-1V$ overlaid ….showing band-bending correctly ---**2Pts***
I am necessarily looking for some sort of indication that the MOSCAP is into inversion

(d) **(4Pts)** Plot the surface potential, φ_s vs. V_g



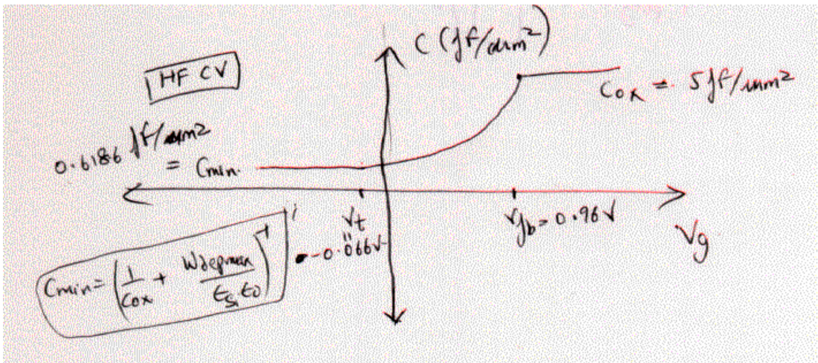
In above figure V_t should be -0.34V
*Correct Plot – **3Pts** (Positive φ_s is OK …provided inversion is for Vg<Vt)*
*Label –**1Pt***

(e) **(4Pts)** Plot the substrate charge, Q_{sub} (= Q_{acc}+Q_{dep}+Q_{inv}) vs. V_g



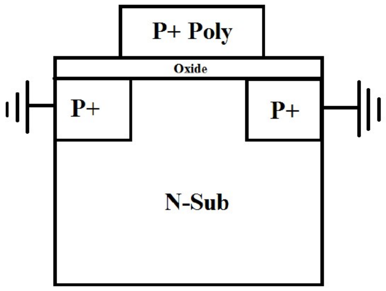
In above figure V_t *should be -0.34V and* $Q_{dep,max} = 1fC/\mu m^2$
*Correct Plot ---**2Pts** (Will not give points if drawn for P-Substrate)*
Basic labels .. V_{fb}, V_t *---**1Pt***
Advance labels … $Q_{dep,max}$ *, slopes etc …--**1Pt***

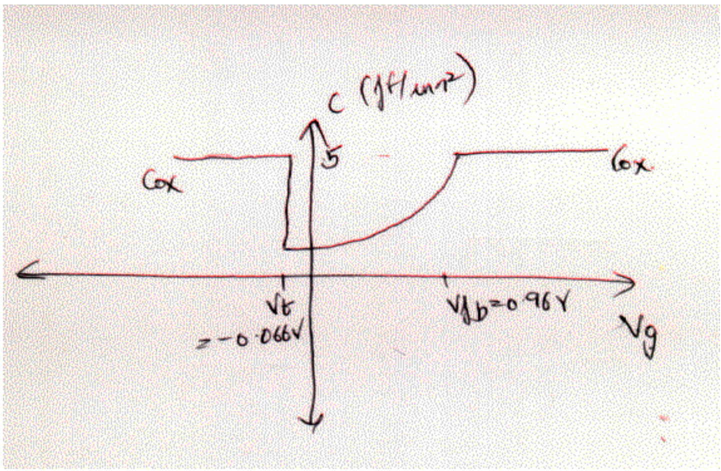
(f) **(4Pts)** Sketch C-V of the MOSCAP at a frequency, f=10MHz.



In above figure V_t *should be -0.34V;* $C_{ox} = 2fF/\mu m^2$ *and* $C_{min} = 0.57\ fF/\mu m^2$
*Correct Plot ---**2Pts** (Will not give points if drawn for P-Substrate)*
Basic labels .. V_{fb}, V_t *---**1Pt***
Advance labels ..C_{ox} *and* C_{min} *etc …--**1Pt** (Not looking for* C_{min} *value)*

(g) **(4Pts)** After applying P+ implantation to form a MOSFET below, plot C-V again at f=10MHz.





In above figure V_t *should be -0.34V;* $C_{ox} = 2fF/\mu m^2$*;* $C_{min}=0.57fF/\mu m^2$

*Correct Plot ---**2Pts** (Will not give points if drawn for P-Substrate)*

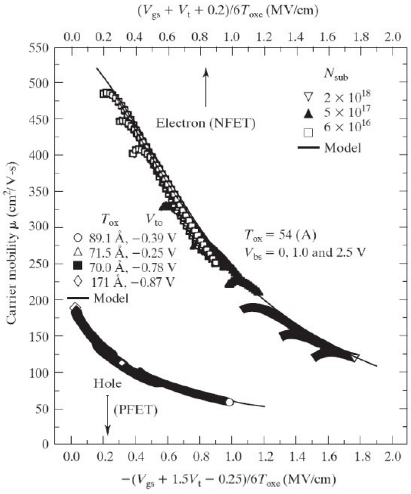
Basic labels .. V_{fb}*,* V_t *---**1Pt***

Advance labels … C_{ox}*,* C_{min} *etc …--**1Pt***

4. MOSFET I-V

Given an N-Channel MOSFET of W=100μm, L=1μm, $V_{t0}=0.5V$, W_{dep_max} = 30nm, T_{oxe}=5nm, $V_{gs}=2.0V$.

(a) **(4Pts)** What is the value of channel mobility, μ_{ns}? (You may need to consider mobility degradation)



Textbook formula estimate

$$\mu_{ns} = \frac{540cm^2/Vs}{1+\left(\dfrac{V_{gs}+V_{to}+0.2}{5.4*T_{oxe}}\right)^{1.85}} = 270cm^2/Vs$$

The term in the parenthesis should have units of MV/cm

Estimate from graph is also acceptable
*Any acceptable estimate –**4Pts***

(b) **(4Pts)** Determine the source to drain current, I_{ds} at $V_{ds}=0.8V$?

$V_{dsat}=(V_{gs}-V_{t0})/m = (2.0 - 0.5)/1.5 = 1.0V > V_{ds}=0.8V$ => Linear region

$$m = 1+3.T_{oxe}/W_{d\max} = 1+15/30 = 1.5$$

$$I_{ds} = \frac{W}{L}C_{oxe}.\mu_{ns}.(V_{gs}-V_{t0}-\frac{m}{2}V_{ds})V_{ds} = \frac{100}{1}\times\frac{\varepsilon_0.\varepsilon_{ox}}{T_{oxe}}\times 270\times 0.9\times 0.8 = 13.42mA$$

*Calculating 'm' ---**1Pt**, Identifying Linear region – **1Pt**, Ids formula and value – **2Pts***

(c) **(4Pts)** What is the current, I_{ds} at $V_{ds}=2.0V$?

$V_{dsat}=(V_{gs}-V_{t0})/m = (2.0 - 0.5)/1.5 = 1.0V < V_{ds}= 2.0V$ => Saturation region

$$m = 1+3.T_{oxe}/W_{d\max} = 1+15/30 = 1.5$$

$$I_{ds} = I_{dsat} = \frac{W}{2mL}C_{oxe}.\mu_{ns}.(V_{gs}-V_{t0})^2 = \frac{100}{3}\times\frac{\varepsilon_0.\varepsilon_{ox}}{T_{oxe}}\times 270\times 1.5\times 1.5 = 13.98mA$$

*Identifying Saturation region – **2Pts**, Ids formula and value – **2Pts***

(d) **(4Pts)** Determine the threshold voltage, V_t when the body-source junction is reverse-biased by 1.0*V*?

V_t (V_{sb}=1V)= V_{t0} + (m-1)V_{sb} = 0.5 + 0.5*1 = 1V*
*Using correct formula – **2Pts**, Calculation -**2Pts***

(e) **(4Pts)** What is the mobility, μ_{ns} under the new condition in (d)?

Textbook formula estimate

$$\mu_{ns} = \frac{540cm^2/Vs}{1+\left(\frac{V_{gs}+V_t+0.2}{5.4*T_{oxe}}\right)^{1.85}} = 227.95cm^2/Vs$$

Note the use of V_t instead of V_{t0}
Estimate from graph is also acceptable
*Identifying usage of V_t instead of V_{t0} –**2Pts**, Estimate of mobility – **2Pts***

(f) **(5Pts)** Calculate I_{ds} at V_{ds}=0.8*V* for this new condition?

V_{dsat} = (V_{gs} – V_t)/m= (2 -1)/1.5 =0.667V < V_{ds} = 0.8V => Saturation region

$$I_{ds} = I_{dsat} = \frac{W}{2mL}C_{oxe}.\mu_{ns}.(V_{gs}-V_t)^2 = \frac{100}{3}\times\frac{\varepsilon_0.\varepsilon_{ox}}{T_{oxe}}\times 270\times 1.0\times 1.0 = 6.2133mA$$

*Identifying Saturation region – **3Pts** , I-V ---**2Pts***