Saturday, December 15, 8:00–11:00 AM, 2001.

Answer all questions for a maximum of 100 points. Please write all answers in the space provided. If you need additional space, write on the back sides. Indicate your answer as clearly as possible for each question. Write your name at the top of each page as indicated. *Read each question very carefully!*

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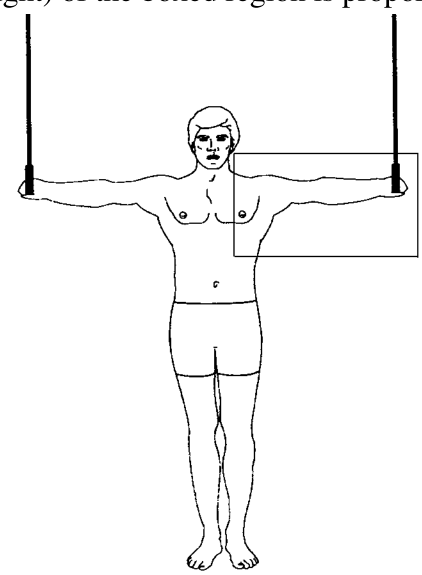
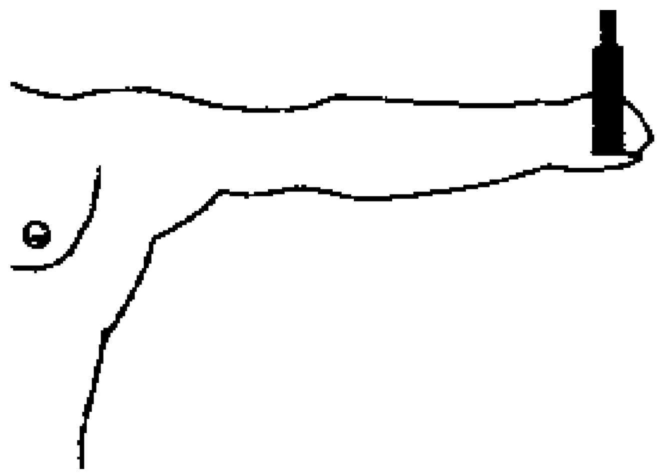
## 1. Biomechanical Analysis of the Skeleton [15 points total]

The picture below (left) shows a gymnast performing the "iron cross" routine, in which he is suspended as shown by the ropes in each hand. Assuming that only one muscle group is dominant, and using a vector force triangle approach, estimate the magnitude of the joint contact force at the shoulder for this routine in terms of the body-weight of the gymnast. State any additional assumptions. The zoomed-in view (below, right) of the boxed region is proportionally to scale.





## 2. Contact Stresses and Design of Shoulder Prostheses [20 points total]

**A.** The figure below shows a schematic of a total joint prosthesis for the gleno-humeral joint, with the main design parameters: radius of the humeral $R_h$ and glenoid $R_g$ components, as well as the average thickness of the plastic, $t$. Indicate whether the following statements are true or false:

*The maximum compressive contact stresses will increase if:*

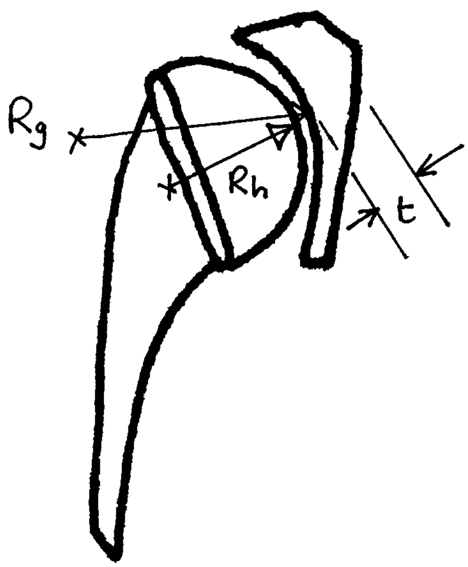
a) $R_h$ is increased (all else constant)

b) $R_g$ is increased (all else constant)

c) $t$ is increased (all else constant)

d) the elastic modulus of the glenoid (plastic) component is increased (all else constant)

e) the elastic modulus of the humeral component is increased (all else constant)

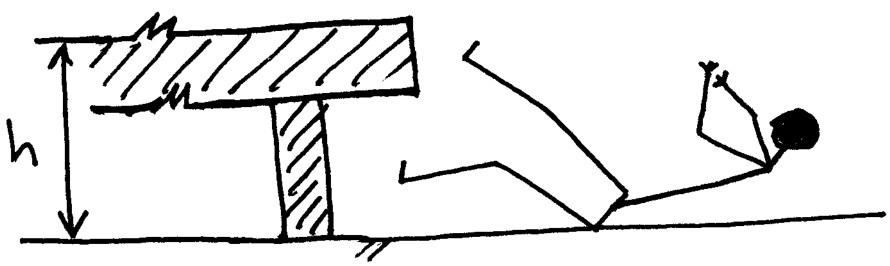


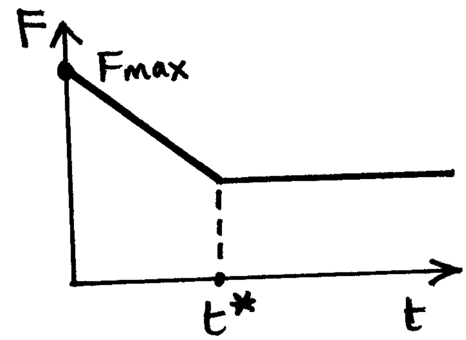

**B.** Will the contact stresses be more sensitive to small (say ± 1 mm) changes in $R_h$ or $R_g$? Explain.

**C.** State two pros and two cons of using metal-backing of plastic components in total joint replacement.

**3. Impact Mechanics and Hip Fracture [20 points total]**
The fall condition is an important factor in hip fracture etiology. In particular the height of the fall can be crucial. To appreciate this, work the following problem.

A. As shown in the figure below, an elderly individual has just fallen off their bed, from a height *h*. They impacted their left hip at the trochanter. Assume the body can be modeled as a point mass. Determine the magnitude of the maximum force at impact $F_{max}$ for this event assuming further that the force-time profile — had there been a force plate placed right at the point of impact — is as shown. Express your answer in terms of the individual's body mass *m*, the height of the bed *h*, the time at the end of impact *t**, and any other parameters that are necessary. State any additional assumptions that need to be made.



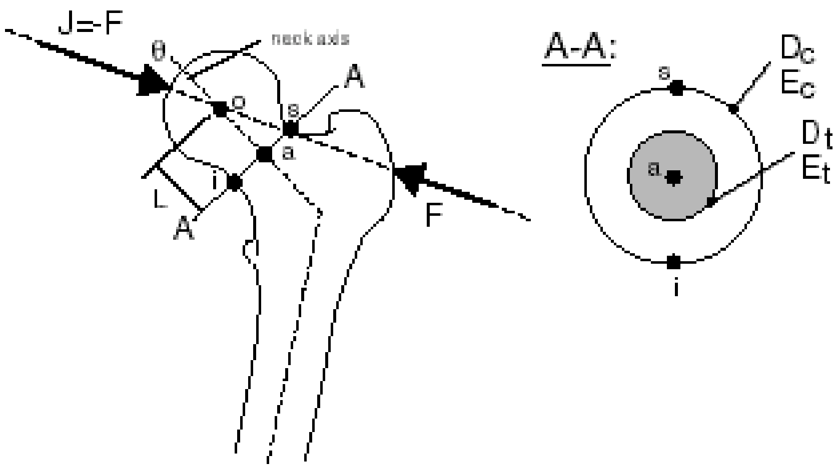
B. Let's think about some practical implications here. If this person had being wearing a hip pad (soft padding strapped to the side of their hip), how could this have changed the maximum force? What about if they were sleeping on a futon? What type of individual is most at risk? Interesting questions….Write a page on how one could — practically — reduce the risk of hip fracture focusing on factors related to the fall. Use sound engineering arguments as part of your rationale.

**4. Composite Beam Theory, Optimization and Hip Fracture Risk [25 points total]**

Fall conditions, specifically the angle of impact of the femur with respect to the impact force *F*, can also play an important role in hip fracture etiology. Consider the situation in which the impact force at the side of the hip during a fall is *F*, and that it develops an equal and opposite joint contact force *J*, as shown below. For a given bone structure, show that the equation given below for *tan*θ specifies the angle θ of the force with respect to the neck axis at which the strength of the femoral neck (at section A-A) will be maximized. Assume concentric arrangement of the trabecular and cortical bone in the cross-section, as shown. Assume also that the ratio of tensile to compressive yield stresses of the cortical bone is 0.7, and that the cortical bone will fail before the trabecular bone, *i.e.* the neck as a structure will fail when stresses in the cortical bone reach the yield stress. Assume linear elastic behavior is valid up to the yield point.

*Note: in deriving this formula, there's no need to break out the summation terms.*

$$\tan\theta = \frac{0.353}{LD_c}\frac{\sum EI}{\sum EA}$$



**5. Composite Beam Theory, Surgical Factors, and Implant Design [20 points total]**
Surgical conditions can affect the biomechanical outcome of a bone-implant system sometimes just as much as can traditional design variables. The figure below shows a cross-section of an idealized cemented hip stem in the mid-diaphysis, showing two extreme mediolateral surgically-placed positions of the stem with respect to the bone, and the perfectly central position. Bending is about some vertical axis parallel with the Y-axis. *Without using any mathematical equations*, suggest which of these configurations would have the lowest risk of fatigue failure of the stem. Provide a qualitative but nonetheless detailed rationale.