Wednesday, December 15, 12:30–3:30 PM, 1994.

- Write all answers in the space provided. If you need more space, write on the back sides.
- Indicate your answer as clearly as possible for each question.
- Write your name on the top of each page as indicated.

1. (10 points) Forces and Moments at Joints

Figure 1 depicts a schematic of the proximal femur with the joint contact *J* and abductor *A* forces shown, during the single legged stance phase of gait. Show, with your own free-body diagram analysis, why the angle □must always be greater than angle □.

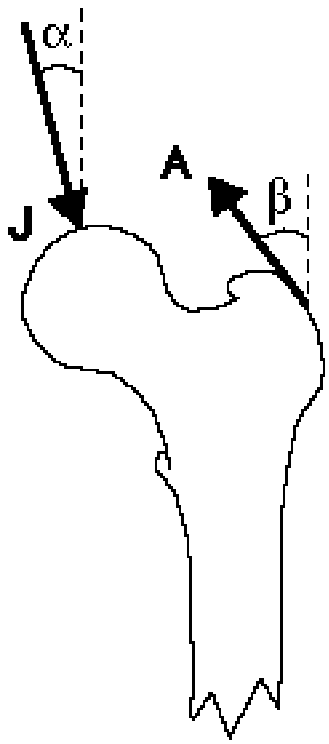
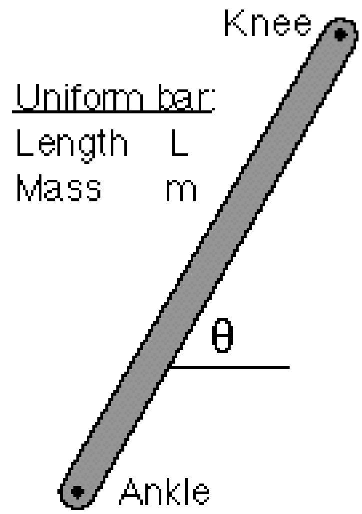


Figure 1

2. (15 points) Dynamics of Human Motion

Figure 2 depicts a schematic of the shank (knee-ankle segment of the leg), at an angle θ to the horizontal, that can be used to compare the resultant moment about the knee joint for heel-strike vs. the swing portions of gait. For the following two cases, draw a free-body diagram showing all loads and accelerations and write out the equation of rotational equilibrium as specified below. In each case, ignore the mass of the foot, model the shank as a uniform bar of length *L* and mass *m*, and include all relevant inertial (acceleration) terms. Label clearly your free-body diagrams and all inertial terms, particularly the moment-of-inertia terms. For loads, show resultant joint reaction forces and moments (as opposed to joint contact and muscle forces).



Note: $I_{cg} = 1/12\ mL^2$

Figure 2

(i) Heel strike. Here, the ground reaction force acts through the ankle and the ankle joint is fixed in space (*i.e.* model the shank as an inverted pendulum). For your rotational equilibrium expression, take moments about the *ankle* joint.

(ii) Swing phase. Here, there is no ground reaction force and there is no fixed point. For your rotational equilibrium expression, take moments about the *knee* joint.

3. (10 points) Joint Stability

(i) Describe briefly the primary and two secondary stability mechanisms that can provide moment equilibrium in the frontal plane at the knee joint when a medially-directed component of the ground reaction force acts on the foot. For each case, comment on the effects of increasing force and increasing loading rate on the applicability of the mechanisms.

A. Primary:

B. Secondary:

C. Secondary:

(ii) What relevance do these mechanisms have on the design of total joint prostheses?

4. (15 points) Mechanical Behavior of Musculoskeletal/Orthopaedic Materials

(i) Sketch tensile stress-strain curves (on the same labeled graph) for wet cortical bone for longitudinal and transverse loading. Indicate typical stress and strain values on each axis.

(ii) Fill in the following table of material properties with approximate values (for blank cells only):

Material	*Young's modulus (MPa)*	*Compressive yield strength (MPa)*	*Tensile yield strength (MPa)*
Trabecular bone †			—
Cortical bone ††			
Tendon		—	—
Articular cartilage §		—	—
UHMWPE		—	
PMMA			
Ti-6Al-4V alloy		—	
CoCr alloy (forged)		—	

† for trabecular bone, give typical maximum values for human tissue
†† for cortical bone, give values for longitudinal loading
§ for cartilage, give equilibrium value

(iii) What is the relationship between apparent density ρ_a, tissue density ρ_t, and volume fraction V_f for trabecular bone?

(iv) An increase in percentage proteoglycan content of articular cartilage will (indicate the correct response) increase, decrease, or not change the

tensile equilibrium modulus:

compressive equilibrium modulus:

(v) Sketch Hill's spring-dashpot-actuator "active state" model for muscle, labeling all elements:

5. (20 points) Beam-on-Elastic-Foundation Theory

Derive from first principles the following fourth order differential equation for the displacement □ of the neutral axis of a beam of flexural stiffness *EI* that is supported on an elastic foundation of "stiffness" *k* , and loaded with a force per unit length *p*. State clearly and explicitly all your assumptions.

$$\frac{\partial^2}{\partial x^2}\left\{EI\ \frac{\partial^2 \square}{\partial x^2}\right\} + k\square = p$$

Hints: • Start with an equilibrium analysis of an element of the beam of length *∂x*...

• For bending of a beam with "small" deformations of the neutral axis □, the following approximation holds where □ is the radius of curvature of the deformed neutral axis:

$$\frac{1}{\square}\ \square\ \frac{\partial^2 \square}{\partial x^2}$$

Question 5, continued

6. (15 points) Design of Hip Prostheses

(i) Write out an expression for the maximum tensile stress in the stem portion of a cementless hip prosthesis for loading by a pure bending moment M and an axial force P. Assume there is ideal load sharing between the stem and bone. Explain your nomenclature.

(ii) State two important assumptions that must be satisfied in order for this equation to be valid.

A.

B.

(iii) Based on this equation, plot a typical graph of stem stress (Y-axis) vs. stem diameter (X-axis):

(iv) Based on this equation, plot a typical graph of stem stress (Y-axis) vs. periosteal bone diameter (X-axis):

(v) If the stem length were shortened substantially, would you expect composite beam theory to still apply? Explain briefly your answer.

7. (15 points) Design of Knee Prostheses

(i) On the same graph, plot the maximum contact stress (Y-axis) vs. plastic thickness (X-axis) to illustrate the different qualitative behaviors of the following cases: (A) highly conforming surfaces; (B) poorly conforming surfaces; and (C) poorly conforming surfaces with increased modulus of the plastic component. Uses the labels A, B, and C to distinguish the different curves.

(ii) Give three implications of this graph in terms of the design of knee prostheses:

A.

B.

C.

(iii) Give one advantage and two disadvantages of using metal backing on the plastic component of total joint components:

Advantage:

Disadvantage:

Disadvantage:

Additional work space for miscellaneous questions (indicate question):