MIDTERM EXAM

BioE 153

2:00 pm, November 13, 2003

NAME: Solution

ID NUMBER:

Prob. 1	Prob. 2	Prob. 3	Prob. 4	Total
Max = 30	Max = 25	Max = 25	Max = 20	Max = 100

Remember to work the problems out clearly and completely – so that I can understand what you are doing.

The problems are designed so that you can answer them all in the space below the problem.

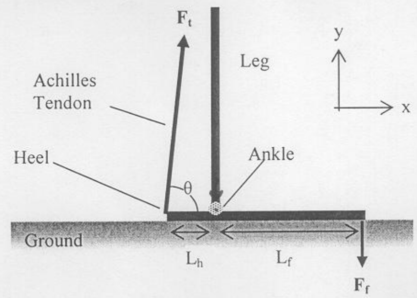
Here are some equations that may be helpful.

Poissons ratio: $$\upsilon = \frac{\varepsilon_{transverse}}{\varepsilon_{longitudinal}}$$

Shear Stress in a fluid flow $$\tau = \mu \frac{du}{dy}$$

1. Find the forces on your ankle joint as you lift your heel just off the ground. Assume that your foot can be modeled as a single rigid structure and that your ankle can be modeled as a single hinge. You can also assume that your leg is vertical and that the downward force at the front of your foot is straight down.

F_t = 2-D force of your Achilles tendon
F_f = 2-D force of your foot
m = your mass
θ = the angle of your Achilles tendon
L_f = the length of your foot in front
L_h = the distance between the joint and the tendon



A. Find the components of force (F_x and F_y) on your ankle joint.

B. What happens to the joint forces if you bend forward at your ankle?

(5) $\sum_1' \bar{F} =$ statics

$\sum_1' \bar{M} = 0$

$\boxed{\text{no moments} \Rightarrow -10}$

(5) $\sum F_x = F_t \cos\theta$

$\sum F_y = F_t \sin\theta + F_f$ ~~mg~~

(5) $\sum_1' \bar{M}_{2\,\text{ankle}} = 0 = -L_H F_t \sin\theta - L_f F_f$

(5) $\boxed{F_f = -mg}$

$L_H F_t \sin\theta = L_f mg$

(5) $\Rightarrow \boxed{F_t = \frac{L_f}{L_H}\frac{mg}{\sin\theta}}$

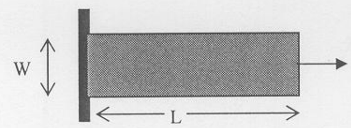
$F_x = F_t \cos\theta = \frac{L_f}{L_H} mg \cancel{\sin\theta} \frac{\cos\theta}{\sin\theta} = mg\left(\frac{L_f}{L_H}\frac{\cos\theta}{\sin\theta}\right)$

$F_y = F_t \sin\theta - mg = \frac{L_f}{L_H}\frac{mg}{\cancel{\sin\theta}} - mg = mg\left(\frac{L_f}{L_H}\cancel{\frac{1}{\sin\theta}} - 1\right)$

(5) B. θ decreases ⇒ joint forces increase when you lean forward

$F_y \uparrow$ as $\cos\theta \to 1$, $F_x \sim$ const.

2. Consider a rubber bar in 2-dimensions that you magically pull from one side as shown in the figure.



A. If you pull the end a small distance, δ, what happens to the width of the bar assuming that the Poisson's ratio, ν, is greater than zero?

B. Assume that the rubber is incompressible, show that ν = ½.
- First, write down what it means for the material to be incompressible,
- Second, draw a picture of the bar when you pull on it, and then
- Third, Solve the problem for displacements that are very small (.δ/L << 1)

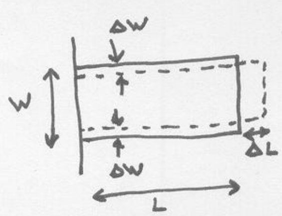
$\nu = \frac{\varepsilon_t}{\varepsilon_l}$ $\varepsilon_L = \frac{\Delta L}{L}$

$\varepsilon_t = \frac{2\Delta w}{w}$ ← Δw total

5 A. w has to decrease

5 B. Incompressible for 2-D ⇒ the area is constant $W \cdot L = (W - 2\Delta W)\cdot(L + \Delta L)$

5



5 $\cancel{\delta} = \cancel{\frac{\Delta L}{L}} \cdot \cancel{\frac{w}{2\Delta w}}$

$\nu = \frac{2\Delta W}{W} \frac{L}{\Delta L}$

$W \cdot L = W \cdot L + \Delta L \cdot W - 2\Delta W L - \underbrace{2\Delta W \cdot \Delta L}_{\sim 0}$ 5

$2\Delta W L = \Delta L \cdot W$

$\frac{1}{2} = \frac{\Delta W}{W} \cdot \frac{L}{\Delta L} = \nu$

3. In artificial liver systems, researchers are trying to attach liver cells to the walls of rectangular flow channels through which blood can be pumped to clean out toxins. One of the big problems is to make the channels small so that there is a lot of surface area for the cells while still moving the flow through at a reasonable rate. This increases fluid forces on the cells which can tear them off. *Using the simplified model below, consider the forces are on a rectangular liver cell.*

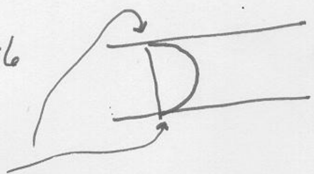
a) Draw the velocity profile in the channel ahead of the cell assuming that the flow is fully developed (i.e. is not changing).

b) Using your drawing, show where the maximum shear stress occurs in the flow?

c) How does increasing the flow rate increase the shear force on the cell?

d) Without solving the problem mathematically, describe how you could ***estimate*** the shear force on the liver cell (1-2 sentences).

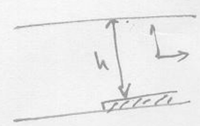
(5) a) parabolic profile



(5) b) τ_{max} @ walls

(5) c) $Q \uparrow \Rightarrow u_{ave} \uparrow \Rightarrow \tau \uparrow$

d) $\tau = \mu \frac{\Delta u}{\Delta y} = \mu \frac{u_{max}}{h/2}$

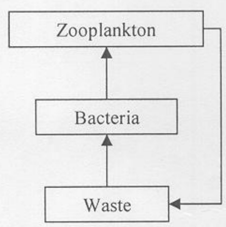


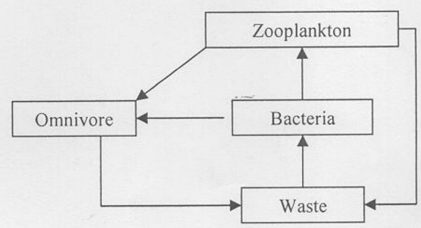
~~Force =~~ $\tau = \frac{F}{Area}$ (5)

(5) $F = \text{Cell area} \cdot \tau \approx \text{cell area} \cdot \mu \frac{u(0)}{h/2}$ ($\approx u_{max}$)

4. Consider the food chain modeled below in the figure on the left. A more complex food web can be created by adding an omnivore (e.g. it eats anything) as shown on the right. Assume that the system is materially closed, i.e. it is in a sealed jar.

    a) Assume that you want to model the dynamics of the closed ecosystem using conservation of mass. What would you model and what would the units of the variables?
    b) How many 1st order differential equations would you have for the food chain?
    c) Assume that you decide to add in an omnivore as shown on the right. Would the resulting system be more stable? Why?





5 a) conservation of mass ⇒ cons. of an element

e.g. Carbon

5 units = $\frac{mg}{l}$ (or $\frac{mols}{l}$)

5 b) 3

c) omnivore keeps zooplankton & Bacteria
5 in balance: if one population grows too much
it is preferentially eaten by the omnivore.

- reduces amplitude of population oscillations.