

MIDTERM EXAM

BioE153

2:00 pm, October 21, 2004

NAME: Solution

ID NUMBER: _____

DISCUSSION SECTION _____

Prob. 1	Prob. 2	Prob. 3	Prob. 4	Total
Max = 30	Max = 20	Max = 30	Max = 20	Max = 100

Remember to work the problems out clearly and completely – so that I can understand what you are doing. Here are some equations that may be helpful but may not be required.

Uniaxial Loading:

$$\sigma = \frac{F}{A}$$

Pure Bending

$$\sigma = \frac{Mt}{I}$$

M = Moment

t = distance from the neutral axis

I = Moment of Inertia

A = Cross Sectional Area

1. (30 pts) Myelodysplastic syndrome is a bone marrow disorder characterized by abnormal bone marrow activity. The disorder is treated by bone marrow transplantation whereby defective bone marrow cells are replaced by progenitor cells, which are cells that have not yet become adult bone marrow cells.

Within a cellular population, a progenitor cell may decide to 1) differentiate – thereby becoming an adult cell, 2) enter what is called apoptosis – cell death, or 3) divide into more progenitor cells through a process called mitosis.

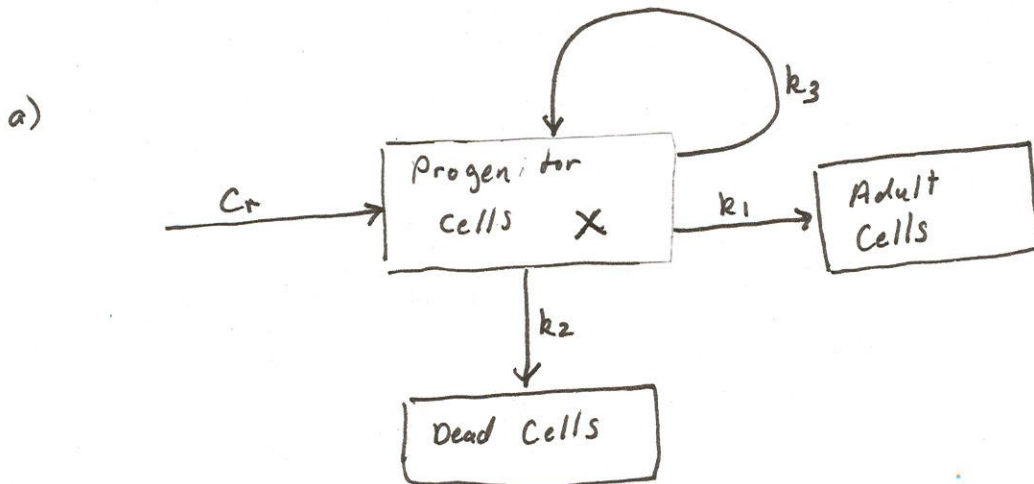
Let C_r be a given number of cell type X introduced into the patient per unit time.

k_1 be the rate constant for differentiation,

k_2 be the rate constant for apoptosis, and

k_3 be the rate constant for mitosis.

- Draw a compartment model describing the possible fate of progenitor cells.
- Derive the differential equation that can mathematically model this process.
- For the transplantation to be successful, we wish the number of transplanted progenitor cells remain constant in the patient. For this to be possible, what must C_r equal?
- Does the solution to your equation in part B resemble drug concentration changes within the body for a rapid bolus or slow infusion drug delivery system? Use a mathematical argument for your answer.



b)
$$\frac{d[X]}{dt} = C_r - k_1[X] - k_2[X] + k_3[X]$$

c) constant progenitor cells $\Rightarrow \frac{d[X]}{dt} = 0$

$$0 = C_r - k_1[X] - k_2[X] + k_3[X]$$

$$\text{or } C_r = k_1[X] + k_2[X] - k_3[X] = [k_1 + k_2 - k_3][X]$$

$$\begin{aligned}
 d) \quad \frac{d[X]}{dt} &= C_r - k_1[X] - k_2[X] + k_3[X] \\
 &= C_r - (k_1 + k_2 - k_3)[X]
 \end{aligned}$$

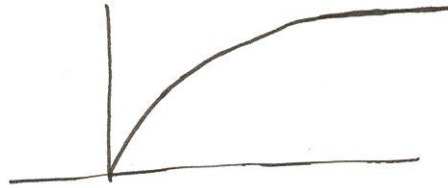
This eqn is of the form

$$[\dot{X}] = C_r - K[X]$$

wt soln $[\dot{X}]$ like $1 - e^{-kx}$



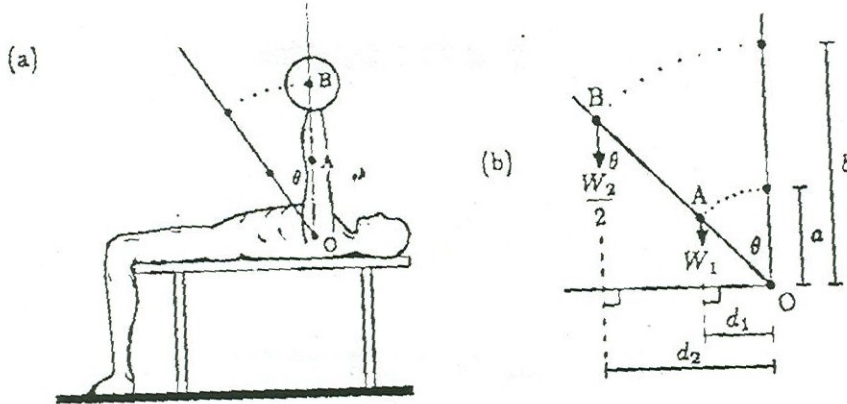
which is the same as for slow infusion



Also you can find C_r for steady state as well.

2. (20 pts) In the figure below left (a), an athlete is doing shoulder exercises by lowering and raising a barbell with her arms straight. The position of her arms, when they make an angle θ with the vertical is simplified in the figure on the right (b).

Determine the net moment, M_o , generated about the shoulder as a function of θ .



~~$\sum \vec{r} \times \vec{F} = 0$~~

$$\begin{aligned}
 M_o &= d_1 W_1 + d_2 \frac{W_2}{2} \\
 &= a(\sin\theta) W_1 + b \sin\theta \frac{W_2}{2} \\
 &= \underline{\underline{\left(aW_1 + b\frac{W_2}{2} \right) \sin\theta}}
 \end{aligned}$$

3. (30 pts) A weightlifter is bent forward and lifting a weight, W_0 , as shown in the left figure. At the position shown, the athlete's trunk is flexed at an angle θ as measured from the upright (vertical) position. You are given the forces acting on the lower portion of the athletes body in middle figure and a simplified model in the right figure.

W is the total weight of the athlete,

W_i is the weight of the legs including the pelvis,

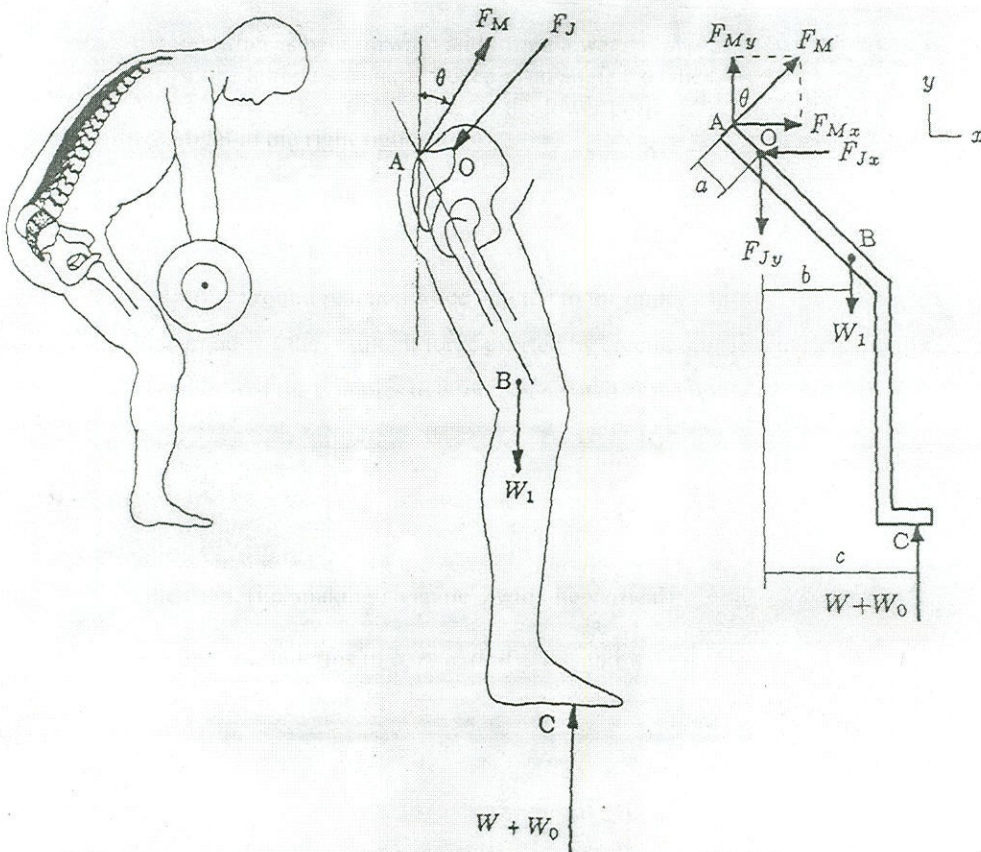
$W + W_0$ is the total ground reaction force applied to the athlete through the feet (at C).

F_M is the magnitude of the resultant force exerted by erector spinae muscle supporting the trunk,

F_J is the magnitude of the compressive force generated at the union of the sacrum and fifth lumbar vertebra.

The center of gravity of the legs including the pelvis is located at B. Relative to O, the lengths of the lever arms of the muscle force lower body weight and ground reaction force are measured as a , b , and c , respectively. The line of pull of the resultant muscle force exerted by the erector spinae muscles is parallel to the trunk (i.e. making an angle θ with the vertical).

Determine F_M and F_J in terms of a , b , c , θ , W_0 , W_i , and W .



Statics

$$\sum_i \vec{M} = 0$$

$$\sum_i \vec{F} = 0$$

$$\sum_i M = 0 \Rightarrow a F_n + b W_1 - c (W + W_0) = 0$$

$$\sum_i F_x = 0 \Rightarrow F_{jx} = F_{nx}$$

$$\sum_i F_y = 0 \Rightarrow F_{jy} = F_{ny} + W + W_0 - W_1$$

solve for F_n in Moment eqn

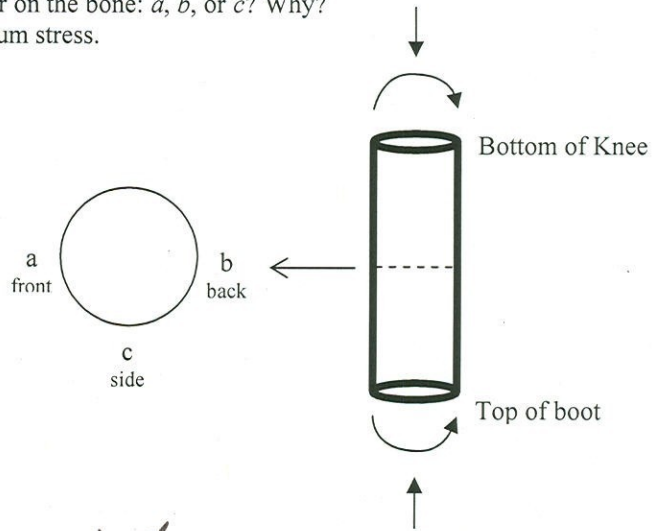
$$F_n = \frac{c(W + W_0) - bW_1}{a}$$

$$F_j = \sqrt{F_{jx}^2 + F_{jy}^2}$$

4. (20 pts) In skiing the lower bone in the leg undergoes significant loading between the bottom of the knee and the top of a ski boot. Based on the figure below, determine the maximum stress and its location on the bone given that the knee successfully stays together and fixed as Dorian lands on the ground so that there is a downward force equal to my body weight.

Assume that the bone is homogeneous and has a Moment of Inertia, I , and Diameter, D .

- A. Where does the maximum stress occur on the bone: a , b , or c ? Why?
 B. Develop an expression for the maximum stress.



A. at b where you have both compressive forces from a downward force as well as from the bending moment

B. $\sum \sigma_i \rightarrow \frac{F}{A} + \frac{M t}{I} = \max \sigma$ where $t = b$

$$\frac{F}{A} + \frac{M b}{I} = \sigma_{\max}$$