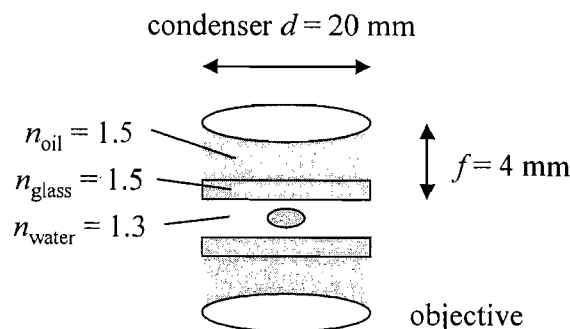


- (10) (1) Write the expression for the wavefunction of a harmonic wave of amplitude 10^3 V/m, period 2.2×10^{-15} s, and speed 3×10^8 m/s. The wave is propagating in the negative x -direction and has a value of 10^3 V/m at $t = 0$ and $x = 0$.
- (20) (2) The light from a 220-W lamp spreads uniformly in all directions. Assuming that 5% of the lamp energy is converted to visible light and you are measuring at a distance 10 meters from the lamp, find
- (a) the intensity of the visible electromagnetic wave (in units of W/m^2) and
 - (b) the E-field amplitude.
- (20) (3) A small object is placed 20 cm from the first of a train of three lenses with focal lengths, in order, of 10, 15, and 20 cm. The first two lenses are separated by 30 cm and the last two by 20 cm. Calculate the final image position relative to the last lens and its magnification relative to the original object when
- (a) all three lenses are positive and
 - (b) the middle lens is negative.
- (20) (4) White light traveling parallel to the optical axis is incident on a thin glass lens with a spherical surface.
- (a) Assuming the lens is positive, draw a diagram illustrating the dispersion of a ray of white light through this lens (label red and blue assuming $n_{\text{red}} < n_{\text{blue}}$).
 - (b) If the lens is negative, draw the dispersion in this case.
 - (c) How might these lenses be combined to reduce chromatic aberration?
- (20) (5) An oil immersion condenser lens is used to illuminate a sample in water, as shown below.
- (a) If the condenser lens diameter is 20 mm and the focal length is 4 mm, what is the maximum numerical aperture of the illumination?
 - (b) Assuming that the oil and glass have the same index of refraction, over what range of angles will the illumination pass through the glass-water interface?
 - (c) What is the resulting maximum resolution of the light collected in transmission by the oil immersion objective lens (assume $\lambda = 500$ nm)?



(10)

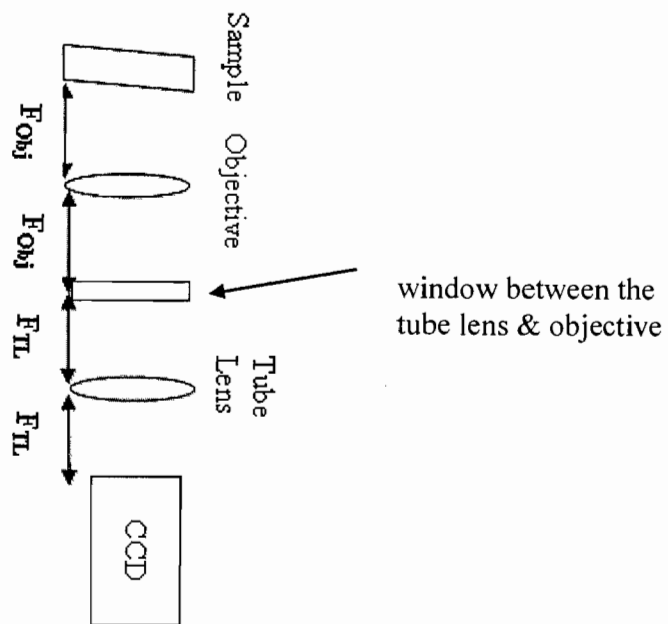
- (6) Like the above optical system, many modern research microscopes are “inverted” – you put the sample above the objective, and all the collection optics are below that. This is convenient if you want to be able to get at your sample from above. Unfortunately, dust falls down... and so can collect on all your optical surfaces.

When you put a sample on the sample stage and focus the objective on it, you discover lots of dirt in focus too. At first you think your sample itself is dirty, but when you move the stage left or right the features in the sample move too but the dirt remains in the same place. Other people in the room see this too, and they offer a few suggestions:

- (a) One person thinks dust got on the microscope objective.
- (b) Someone else thinks the problem may be that dust is getting down into the microscope through an open hole under the objective and landing on the tube lens.
- (c) A post-doc suggests something might have gotten on the camera itself after it was mounted to the microscope.
- (d) A fourth person thinks there may be another surface inside the microscope that has gotten dirty instead of the tube lens – for instance a flat window designed to keep dust out located somewhere between the objective and tube lens.

Conveniently you have taken BioE 164. Thinking on your feet, you explain to everyone where (of the 4 suggestions above) the dirt must be. Where is the dirt, and how do you know it is there and not in the other places suggested?

For reference, the optical path looks like this:



Constants

$$c = 3.0 \times 10^8 \text{ m/s}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m}/\text{A}$$

$$\textcircled{1} \quad \tau = 2.2 \times 10^{-15} \text{ s} \quad \rightarrow \quad \nu = \frac{1}{\tau} = 4.5 \times 10^{14} \text{ Hz}$$

$$v = \nu \lambda = 3 \times 10^8 \text{ m/s} \quad \rightarrow \quad \lambda = \frac{v}{\nu} = 6.6 \times 10^{-7} \text{ m}$$

$$k = \frac{2\pi}{\lambda} = 9.5 \times 10^6 \frac{1}{\text{m}}$$

$$\therefore \psi(x, t) = \left(10^3 \frac{\text{V}}{\text{m}}\right) \cos \left[9.5 \times 10^6 \frac{1}{\text{m}} \left(x + 3 \times 10^8 \frac{\text{m}}{\text{s}} t \right) \right]$$

↖
cos is chosen since $\cos 0 = 1$

$$\textcircled{2} \quad P = (220 \text{ W}) \cdot 0.05 = 11 \text{ W}$$

$$I = \frac{P}{A} \quad ; \quad A = 4\pi r^2 = 4\pi (1.0)^2 = 400\pi \text{ m}^2$$

$$I = \frac{P}{A} = \frac{11 \text{ W}}{400\pi \text{ m}^2} = 8.75 \times 10^{-3} \text{ W/m}^2$$

$$I \equiv \langle S \rangle_T = \frac{c\epsilon_0}{2} E_0^2 \quad \rightarrow \quad E_0 = \sqrt{\frac{2I}{c\epsilon_0}} = 257 \text{ V/m}$$

$$\textcircled{3} \quad (f_1) = 10, \quad (f_2) = 15, \quad (f_3) = 20 \text{ cm}$$

$$s_{o1} = 20 \text{ cm}, \quad \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}, \quad M = -\frac{s_i'}{s_o}$$

$$d_{12} = 30 \text{ cm}, \quad d_{23} = 20 \text{ cm}$$

$$a) \quad f_1, f_2, f_3 > 0$$

$$\frac{1}{s_{i1}} = \frac{1}{f_1} - \frac{1}{s_{o1}} \rightarrow s_{i1} = 20 \text{ cm}, \quad s_{o2} = d_{12} - s_{i1} = 10 \text{ cm}$$

$$\frac{1}{s_{i2}} = \frac{1}{f_2} - \frac{1}{s_{o2}} \rightarrow s_{i2} = -30 \text{ cm}, \quad s_{o3} = d_{23} - s_{i2} = 50 \text{ cm}$$

$$\frac{1}{s_{i3}} = \frac{1}{f_3} - \frac{1}{s_{o3}} \rightarrow s_{i3} = 33.3 \text{ cm}$$

$$M_{\text{total}} = \left(-\frac{s_{i1}}{s_{o1}}\right) \left(-\frac{s_{i2}}{s_{o2}}\right) \left(-\frac{s_{i3}}{s_{o3}}\right) = 2$$

$$b) \quad f_1, f_3 > 0, \quad f_2 < 0$$

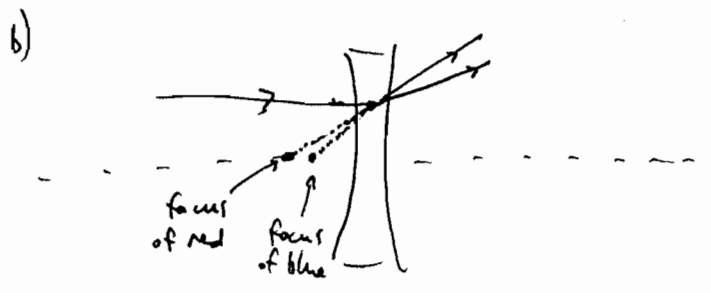
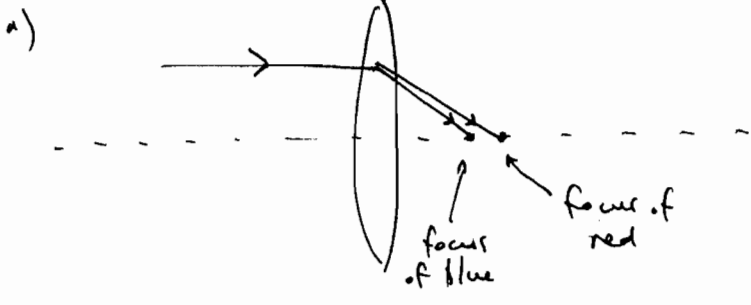
$$s_{i1} = 20 \text{ cm}, \quad s_{o2} = 10 \text{ cm} \quad (\text{same as above})$$

$$\frac{1}{s_{i2}} = \frac{1}{f_2} - \frac{1}{s_{o2}} \rightarrow s_{i2} = -6 \text{ cm}, \quad s_{o3} = d_{23} - s_{i2} = 26 \text{ cm}$$

$$\frac{1}{s_{i3}} = \frac{1}{f_3} - \frac{1}{s_{o3}} \rightarrow s_{i3} = 86.67 \text{ cm}$$

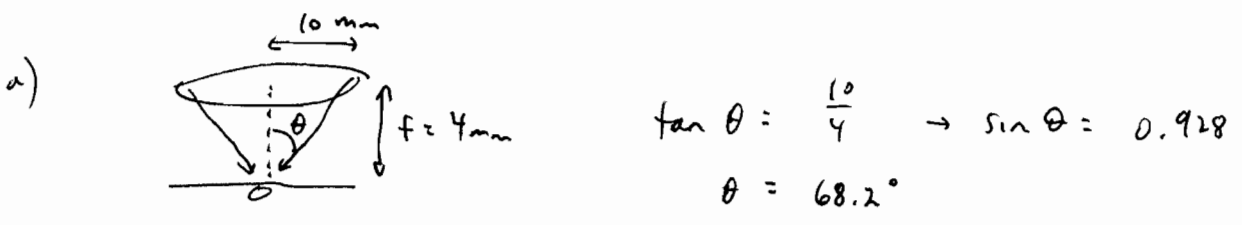
$$M_{\text{total}} = \left(-\frac{s_{i1}}{s_{o1}}\right) \left(-\frac{s_{i2}}{s_{o2}}\right) \left(-\frac{s_{i3}}{s_{o3}}\right) = 2$$

(4)

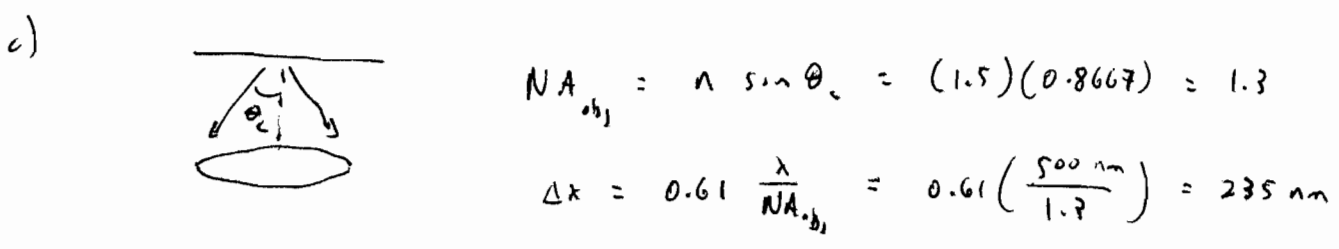
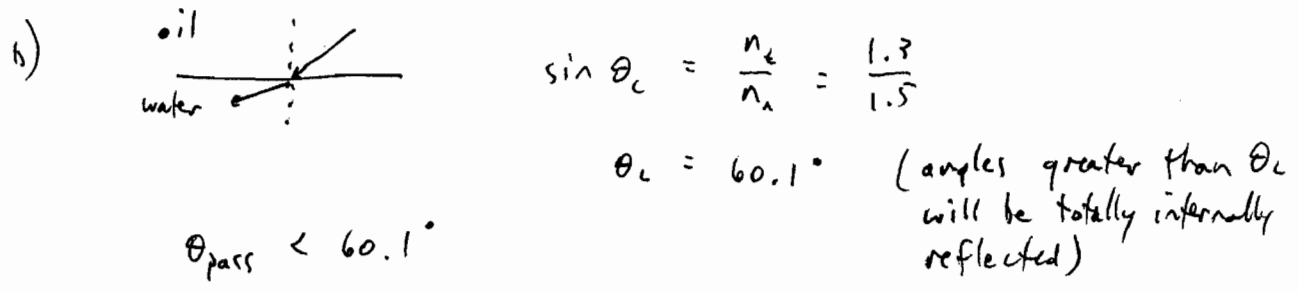


c) a positive and a negative lens could be combined to counteract dispersion and therefore chromatic aberration

(5)



$$NA_{\text{cond}} = n \sin \theta = (1.5)(0.928) = 1.39$$



④ the dirt is on the CCD, since it is the only plane in the optical path that is conjugate with the sample.