Thursday, May 17, 12:30–3:30 PM, 2001.

Answer all questions for a maximum of 100 points. Please write all answers in the space provided. If you need additional space, write on the back sides. Indicate your answer as clearly as possible for each question. Write your name at the top of each page as indicated. *Read each question very carefully!*

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1. Biomechanical Analysis of the Skeleton [15 points total]

A popular web site (http://www.backpain.org/exercise.htm) recommends a number of exercises to help with back problems. One exercise goes as follows: "Stretch one arm forward in front, at the same time stretching the opposite leg out behind." Using a static free body diagram analysis, determine which of either the erector spinae or stomach muscles for this exercise is active (assume only one muscle group is active), and estimate its magnitude in terms of body-weight and any relevant dimensions. State any other assumptions made in your analysis.

2. Design of Knee Prostheses [28 points total]

A. **[5 points]** For the classical Hertz contact problem for contact between two convex cylinders, where are the locations of the:

(i) maximum compressive stress?

(ii) maximum tensile stress?

(iii) maximum shear stress?

Describe a theory that explains how these stress locations may influence the cracking and delamination damage mechanisms in the plastic component of a total knee prosthesis.

B. **[15 points]** Starting with an equilibrium analysis of an element of the beam of length *dx*, derive the following equations for bending of a beam on an elastic foundation. State clearly *all* the assumptions.

i) $$\frac{\partial V}{\partial x} = -p + k\square$$

ii) $$\frac{\partial M}{\partial x} = -V$$

iii) $$\frac{\partial}{\partial x^2}\left\{EI\ \frac{\partial^2 \square}{\partial x^2}\right\} + k\square = p$$

where x is the distance along the length of the beam, V is the shear force acting on the beam, M is the bending moment acting on the beam, p is a uniformly distributed force/length acting on the beam, k is the foundation modulus, EI is the flexural stiffness of the beam with respect to its neutral axis, and $\square = \square(x)$ is the displacement of the neutral axis of the beam.

Hint: For bending of a beam with "small" deformations of the neutral axis $\square$, the following holds:

$$M = EI\,\frac{d^2\square}{dx^2}$$

C. **[4 points]** What is the difference between "stiff" vs. "rigid" behaviors in the context of beam on elastic foundation theory?

D. **[4 points]** Indicate if the maximum contact stress in the artificial knee joint (Figure) should increase or decrease if:

a) R_f is decreased (all else constant)

b) R_t is increased (all else constant)

c) t is increased (all else constant)

d) the modulus of the plastic is decreased (all else constant)

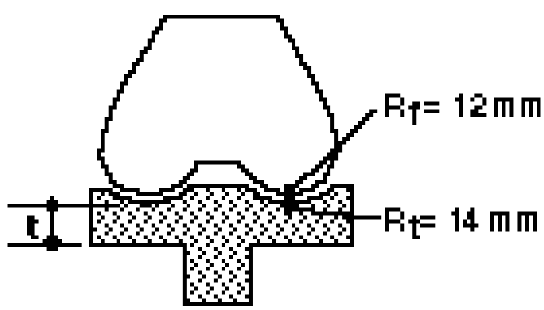


Figure: Frontal view of a bicondylar knee prosthesis. R_f, R_t are the radii of the metal femoral (white) and plastic tibial (shaded) components, respectively; t is the average thickness of the plastic tibial component.

3. Implant Failure Analysis [30 points total]

As an expert biomechanical engineer, you are consulting on a legal case. You have been asked to determine if a material defect was to blame for a failed implant. Here are the facts of the case:

- A large young man (130 kg body mass, 2 m tall, 24 years old) accidentally stepped over a retaining wall while on a ski trip in Lake Tahoe. Upon landing, he felt a sudden pain in his weight-bearing leg. Subsequent radiographs showed a fracture of an intramedullary (IM) rod, half way along its length. The rod had been implanted eight months previously, and was pinned at each end.
- After visiting the site of the accident, you determined that the retaining wall was one meter high. You also found out from the medical records and radiographs that the IM rod had a hollow circular cross-section, inner and outer diameters of 7 and 13 mm, respectively; the corresponding inner and outer diameters of the bone at the cross-section of implant failure were 14 and 28 mm, respectively. The bone appeared fully healed from the previous fracture that had required implantation of the IM rod.
- Upon questioning, the young man revealed that he was wearing expensive Nike sneakers and that the pavement he landed on was concrete. He also noted that he slipped immediately after impact since there was ice on the ground. Since he was not expecting the fall, he was in a gait type upright position at the instant of impact (see Figure). He also mentioned that he was sure he landed on only one foot since his other foot got caught in the retaining wall during the fall.
- Checking with the implant catalogue, you found out that the IM rod was manufactured of cold-worked 316L stainless steel. Looking up your biomaterials textbook, you found that the yield strength of this alloy was 700 MPa and its elastic modulus is 200 GPa.

Discuss, in your expert opinion, why you think the implant failed. As part of your answer, compare the stresses at the site of implant fracture against the yield strength of the implant material.

Hints:

- Use conservation of energy to estimate velocity just before impact; and the impulse-momentum theorem to estimate the peak force at impact. For the latter, assume that the force-time profile at impact, as would be measured if there were a force platform under the young man's foot during the impact, is as shown below (see Figure).
- Use composite beam theory for calculation of stresses. Since the cortical bone in his implanted femur is newly healed bone and therefore not fully mineralized, assume its elastic modulus is 12 GPa. Assume concentric circular cross-sections of the bone and implant.

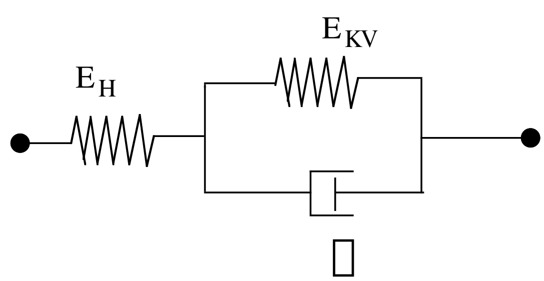
4. Miscellaneous [28 points total]

A. [5 points] Fill in the following table of elastic modulus properties with approximate values:

Material	*Young's modulus (MPa)*
Cortical bone †	
Trabecular bone† Young hip	
Trabecular bone† Old spine	
Tendon	
UHMWPE	
PMMA	
Ti-6Al-4V alloy	
CoCr alloy, cast	
CoCr alloy, forged	
CoCr alloy, HIPed††	

† for bone, give a typical mean value for the longitudinal direction.
†† HIPed — hot isostatically pressed.

B. [15 points] The general time-dependent behavior of many soft tissues, and even bone, can be modeled approximately using spring-dashpot models, such as that shown below.



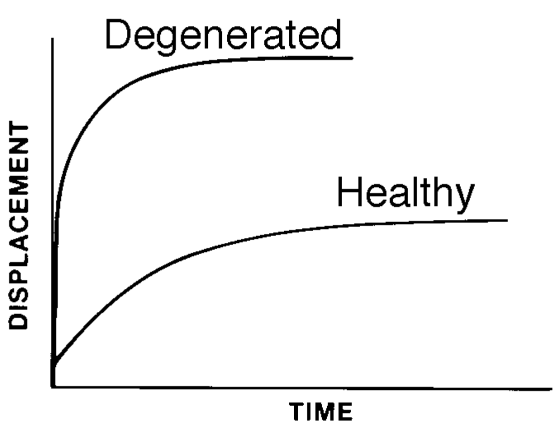
Show that the governing differential equation relating stress □ to strain □ for this model is of the form:

$$\frac{\partial \square}{\partial t} + a\,\square = b\,\frac{\partial \square}{\partial t} + c\,\square$$

by deriving expressions for the constants *a*, *b*, and *c* in terms of E_H, E_{KV}, and □.

C. [4 points] What are the two main structural components of the intervertebral disk?

The Figure below shows a comparison of the creep response to the same static load for a healthy vs. severely degenerated intervertebral disk. Explain how this graph shows that the degenerated disk is less stiff and less viscoelastic than the healthy disk.



D. [4 points] Sketch a typical shear stress distribution along the interface of two concentric cylinders, loaded in compression where all load is transferred from one cylinder to the other via shear through a thin interface. Assume that the overall structural behavior of the cylinders is "flexible", as in the context of shear lag theory, and that the axial stiffness (EA) is larger for the inside component.