Tuesday, May 17, 12:30–3:30 PM, 1994.

Please write all answers in the space provided. If you need additional space, write on the back sides. Indicate your answer as clearly as possible for each question.

1. (10 points) Forces and Moments at Joints

(i) Draw a free-body diagram of the forearm during a curl exercise (with a 20 kg mass in the hand) that could be used to calculate the *resultant* force and moment acting on the elbow joint. Include all accelerations in your free body diagram and ignore the mass of the hand.

(ii) Based on your free-body diagram, write out the moment equation of motion (dynamics equation) about the elbow joint. Assume that the elbow joint is fixed in space.

2. (5 points) Joint Stability

The lateral collateral ligament in the knee is known to support tensile loads when a medially-directed force is quickly applied to the foot. Determine the magnitude of the medially-directed force F at the foot required to rupture the lateral collateral ligament assuming the strength of the ligament is 1500 N? Use the quadriceps and condylar forces and the dimensions as shown in Figure 1 (overleaf).

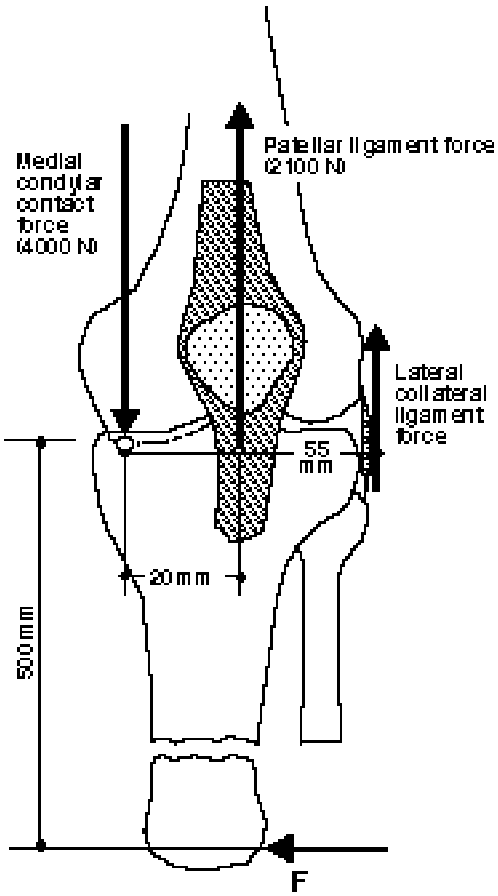


Figure 1

3. (10 points) Trabecular Bone Micro-mechanics
For an open-cell, out-of plane cellular solid (Figure 2) loaded vertically with a stress σ^*, derive the following expressions based on an analysis of elastic deformations and plastic failure of the vertical rods:

i) $E^* = \left(\frac{t}{L}\right)^2 \left[\frac{E_s}{\cos\theta(1 + \sin\theta)}\right]$

ii) $\sigma_y{}^* = \frac{\sigma_{ys} E^*}{E_s}$

iii) If the cell walls have strengths and moduli similar to those of human cortical bone (for longitudinal loading), give a value of $\frac{\sigma^*_{y\text{-tension}}}{\sigma^*_{y\text{-compression}}}$ for this model.

Hint: Assume $\sigma^* = \frac{P}{L^2 \cos\theta(1 + \sin\theta)}$

Nomenclature:

P	internal force acting on each vertical cell wall
*	subscript, referring to properties of the whole cellular solid
s	subscript, referring to properties of the cell wall
σ_y	yield stress
ε_y	yield strain
E	Young's modulus

Figure 2

4. (10 points) Cortical Bone Micro-mechanics
Assuming that τ_{max} and σ_{max} are the shear and tensile strengths of an osteon, respectively, compare the maximum tensile force allowable on a single osteon (embedded length L, diameter D) that fails by:

(i) osteon pullout

(ii) osteon fracture

(iii) If $\tau_{max} = 0.5\ \sigma_{max}$, what is the condition on the osteon aspect ratio (L/D) such that osteon failure will occur first by pullout?

5. (15 points) Composite Beam Theory
Derive from first principles the following expression of the neutral axis for an *n*-material composite beam in pure bending:

$$\hat{y} = \frac{\sum_{i=1}^{n} \overline{y_i}\, A_i E_i}{\sum_{i=1}^{n} A_i E_i}$$

Explain clearly the meaning of each term in this expression. State all major assumptions.
Hint: for pure bending, the sum of all axial forces must be zero.

6. (15 points) Design of Knee Prostheses

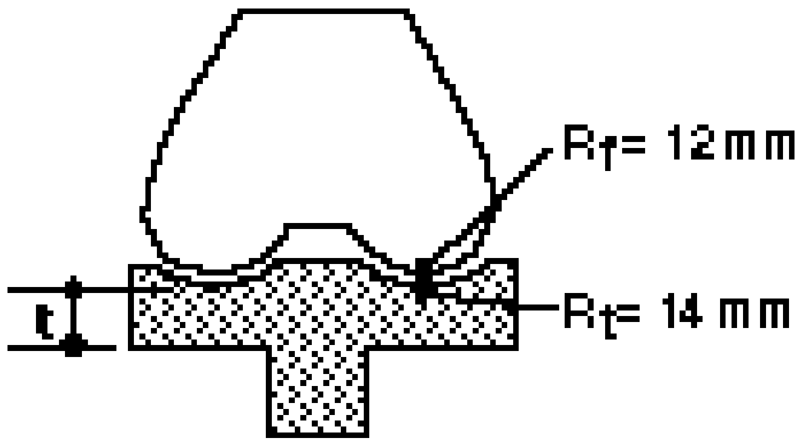
i) Indicate if the maximum contact stress in the artificial knee joint (Figure 3) would increase or decrease if:

a) R_f is increased (all else constant)

b) R_t is increased (all else constant)

c) t is increased (all else constant)

d) the modulus of the plastic is increased (all else constant)



R_f, R_t are the radii of the metal femoral and plastic tibial components, respectively; t is the thickness of the plastic tibial component.

Figure 3

ii) Why are the maximum contact stresses in the plastic usually lower in acetabular than tibial components?

iii) List two disadvantages and one advantage of using metal backing on the plastic component of a total knee replacement?

Disadvantage 1:

Disadvantage 2:

Advantage 1:

7. (15 points) Hip Fracture and Composite Beam Theory

With a fall to the side of the hip, a medially directed force F at the greater trochanter can generate a force J on the femoral head at an angle □ to the femoral axis, as shown in Figure 4. The angle between the neck and femoral axis is □. For this situation:

(i) Express the force J in its components perpendicular and parallel to the neck axis.

(ii) Write out expressions for the maximum tensile stress in the trabecular *and* cortical bone at the cross-section A-A. The neck at A-A is comprised of a circular cortical shell (diameter D_c, modulus E_c) and a concentric circular trabecular core (diameter D_t, modulus E_t). Take the distance from points *o* to *a* as *L*. Express your answer in terms of J, □, □, L, D_t, E_t, D_c, and E_c.

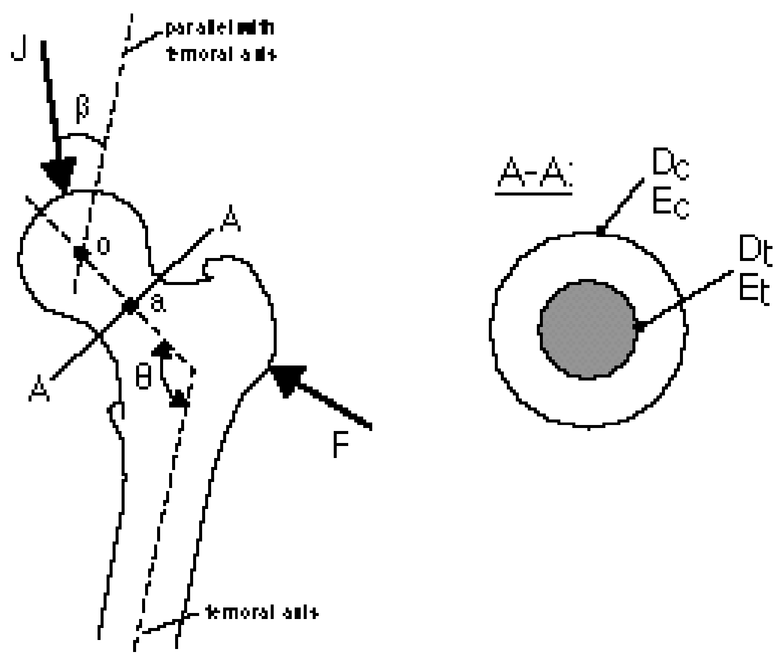


Figure 4

8. (20 points) Miscellaneous

A) Label the following aspects, regions or features of the femur (Figure 5):
medial,
lateral,
distal,
proximal,
greater trochanter,
epiphysis,
diaphysis,
metaphysis,
condyles.

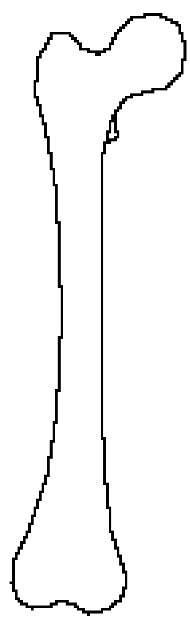


Figure 5

B) Which *one* of the following factors is most associated with hip fractures?
i) bone mineral density
ii) age
iii) fall direction (*i.e.* to the side)
iv) body weight

C) What are the main bone-producing cells?

D) On the same graph, plot stress–strain curves (with strains from 0–50%) for human trabecular bone in uniaxial compression for two different apparent densities; indicate which curve is for the higher apparent density.

E) Cortical bone, as tested in the laboratory (i.e. devitalized), is stronger in compression than in tension, but has better resistance to fatigue fracture for tensile loading. True or false?

F) Name the two metals *and* two polymers that are most commonly used in total joint prostheses. Give an approximate (within 15%) Young's modulus for each of these materials.

G) State one disadvantage of using sintered porous coatings on hip prostheses.

H) On the same graph, sketch typical force–velocity and power-velocity curves for muscle.

I) On the same graph, sketch a tensile stress–strain curve for cortical bone, one for low strain rate loading and one for high strain rate loading; indicate which curve is for the higher strain rate.

J) In reconstructing the hip joint, surgeons sometimes move the point of attachment of the abductors (*i.e*. the greater trochanter) laterally. Will this cause the magnitude of the joint contact force to increase or decrease?