UNIVERSITY OF CALIFORNIA, BERKELEY College of Engineering

Mechanics of Materials (CE130-II)

The First Mid-term Examination (Spring 2004)

Problem 1.

Consider the following statically indeterminate system (Fig. 1). Find the reactions forces R_1 and R_2. Hint: The flexibility is defined as

$$f = \frac{L}{EA}, \tag{1}$$

and relationship between internal force and elongation of a two force bar is $P = f\Delta$.

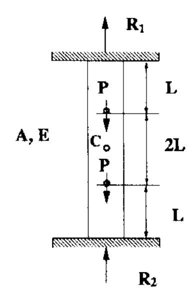


Figure 1: A Statically Indeterminate System

Problem 2

Consider the following two shaft system. Both shafts have circular cross section. Find the maximum shear stress in the system. Assuming $T_B = T$ and $T_C = 2T$. The radius of shaft AB is given as $R = C$; and the radius of shaft BC is given as $R = 2C$. Hints: torsion formula

$$\tau = \frac{T\rho}{I_\rho}, \text{ for shafts with circular cross section }, \quad I_\rho = \frac{\pi R^4}{2}. \tag{2}$$

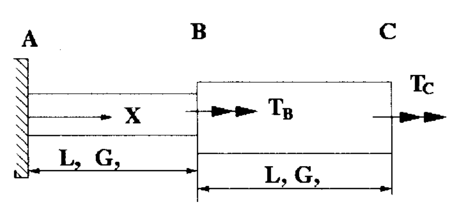


Figure 2: Torsion of a two-shaft system

Problem 3

A planar circular three-hinge arch consists of two segments as shown in Fig. 3. Determine the reaction forces at A and B caused by the application of a vertical force P.

Problem 4

Consider a long (1000 meters in z-direction) concrete block with its both ends fixed. The cross section of the concrete block (section in x-y plane) is a 5 meter square. Suppose that in x-y plane,

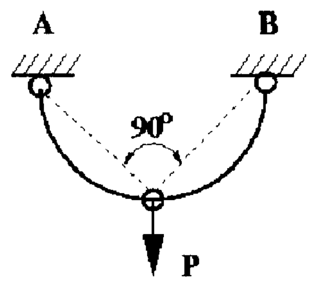


Figure 3: A two-bar truss system

the block is subjected biaxial tensile stress load, namely, $\sigma_x = 5MP_a$ and $\sigma_y = 10MP_a$. This is a typical *plane strain* state. Let $E = 100MP_a$ and Poisson's ratio $\nu = 0.3$. Find σ_z, ϵ_x, and ϵ_y. Hint: The generalized Hooke's law is

$$
\begin{aligned}
\epsilon_x &= \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \\
\epsilon_y &= -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \\
\epsilon_z &= -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}
\end{aligned}
$$

Problem 5.

Consider a rectangular block with the dimension $dx \times dy \times dz$. Uniform shear stress, τ_{xy}, is acting on the surfaces normal to $(+/-)$ x-axis and uniform shear stress, τ_{yx}, is acting on the surfaces normal to $(+/-)$ y-axis as shown in Figure 4. Show $\tau_{xy} = \tau_{yx}$. Hint: use moment equilibrium equation about the z-axis ($\sum M_z = 0$).

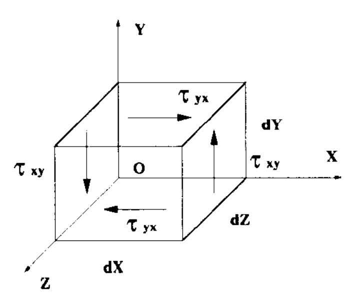
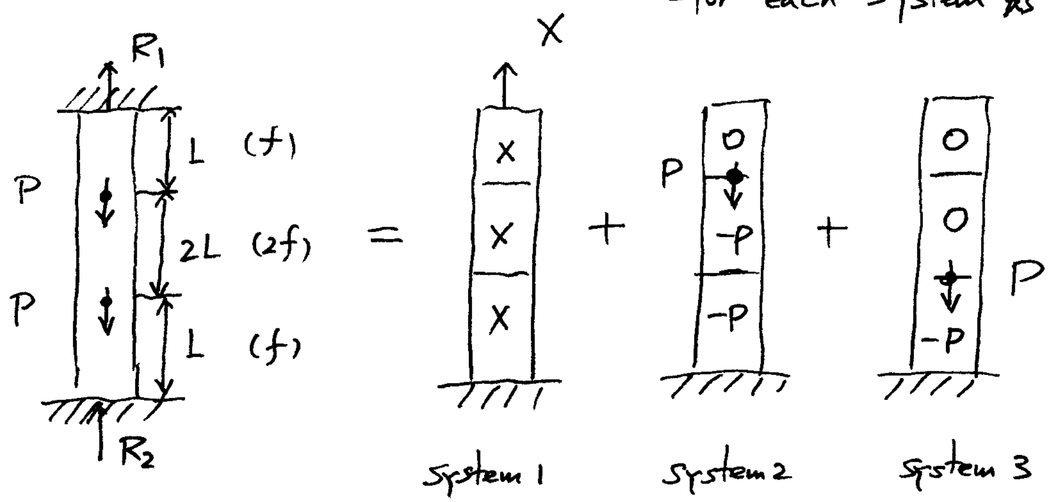


Figure 4: Infinitesimal element in pure shear

Solutions for practice Mid-term Exam

Problem 1. Use superposition method: We first mark the internal force for each system as follow:



then

$$\Delta^{(1)} = fX + 2fX + fX = 4fX$$

$$\Delta^{(2)} = 0 - 2fP - fP = -3fP$$

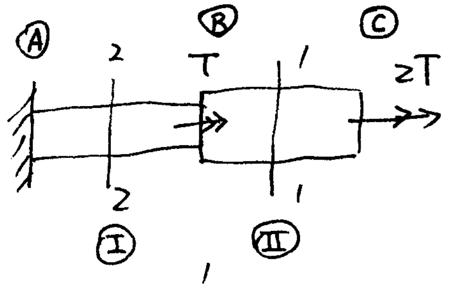
$$\Delta^{(3)} = 0 + 0 - fP = -fP$$

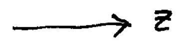
$$\Delta = \Delta^{(1)} + \Delta^{(2)} + \Delta^{(3)} = 4fX - 4fP = 0$$

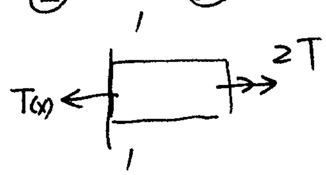
$$\boxed{X = P = R_1}$$

Problem 2

We first find internal torque diagram :



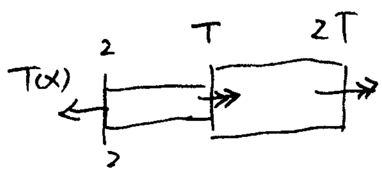




$\Sigma M_z = 0$

$-T(z) + 2T = 0$

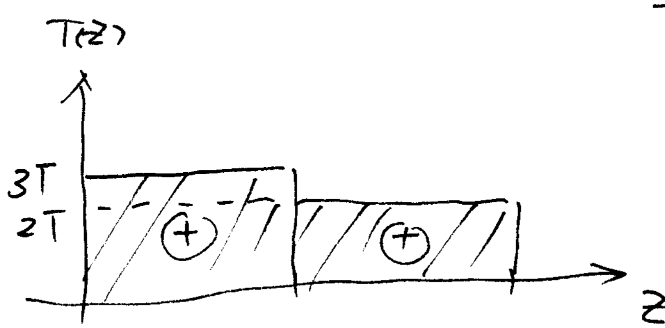
$T(z) = 2T, \qquad L < z \leq 2L$



$\Sigma M_z = 0$

$-T(z) + T + 2T = 0$

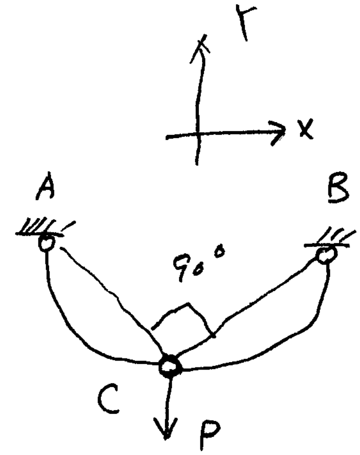
$T(z) = 3T, \qquad 0 \leq z \leq L$



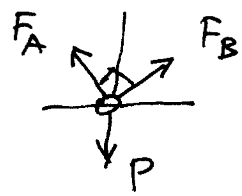
$$\tau_{max}^{AB} = \frac{3T \cdot C}{(\pi C^4)/2} = \frac{6T}{\pi C^3}, \qquad \tau_{max}^{BC} = \frac{2T \cdot 2C}{\pi (2C)^4/2} = \frac{T}{2\pi C^3}$$

The maximum shear stress is : $\boxed{\frac{6T}{\pi C^3}}$

Problem 3



The free-body diagram of the joint is (since both AC & BC are two-force members)



By symmetry : $F_A = F_B$

$$\Sigma F_Y = 0$$

$$F_A \cos 45^\circ + F_B \cos 45^\circ - P = 0$$

$$\sqrt{2} F_A = P \quad \Rightarrow \quad \boxed{F_A = F_B = \frac{P}{\sqrt{2}}}$$

Problem 4.

(1) Plane strain $\varepsilon_{zz} = 0$

$$\Rightarrow \quad -\nu \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{YY}}{E} + \frac{\sigma_{zz}}{E} = 0$$

$$\sigma_{zz} = \nu (\sigma_{xx} + \sigma_{YY}) = 0.3 \times (5 + 10) = 0.3 \times 15 \quad \text{MPa}$$

$$= 4.5 \quad \text{MPa}$$

(2)

$$\epsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_Y}{E} - \nu \frac{\sigma_z}{E} = \frac{1}{E} (5 - 0.3 \times 10 - 0.3 \times 4.5)$$

$$= 0.65 \times 10^{-8} = 0.0065 \quad \mu \text{ strain}$$

$$\epsilon_{YY} = \frac{1}{E}(-\nu\sigma_{XX} + \sigma_{YY} - \nu\sigma_{ZZ})$$

$$= (-0.3 \times 5 + 10 - 0.3 \times 4.5) \times 10^{-8} = 7.15 \times 10^{-8}$$

$$= 0.07 \ \mu \text{ strain}$$

Problem 5

Take a moment around z-axis

$$\Sigma M_{oz} = 0 \quad +\circlearrowleft$$

$$\underbrace{\tau_{XY}\, dz\, dY}_{F_Y} \cdot \underbrace{dx}_{arm} - \underbrace{\tau_{YX}\, dx\, dz}_{F_X}\ \underbrace{dY}_{arm} = 0$$

$$\boxed{\tau_{XY} = \tau_{YX}}$$