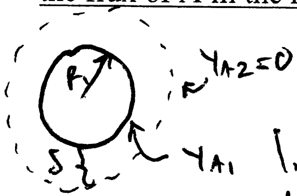
Midterm I - Fall 2005 Solutions

Problem 1 (55 points total)

A volatile liquid droplet (where the liquid is species A) of radius R_1 is suspended in a flowing stream of gas B at steady-state at constant temperature and constant pressure. We postulate that there is a spherical gaseous film consisting of stagnant gas B of radius $(R_1 + \delta)$ surrounding the droplet. The mole fraction of A in the gas phase, y_A, is y_{A1} at $r = R_1$. Also, y_A is 0 at the outer edge of the film at $r = (R_1 + \delta)$. The radius of the droplet is changing extremely slowly as the liquid evaporates into the flowing stream, so you may assume R_1 is constant.

(a) (15 Points) Draw a picture of the system. Starting with a differential shell balance, write out the governing differential equation for mass transfer in the gaseous film in terms of the flux of A in the radial direction, N_{Ar}.



$$\text{In} - \text{out} + \underbrace{\cancel{\text{gen}}}_{\text{no rxn.}} = \underbrace{\cancel{\text{Acc}}}_{0\ \text{- steady state}}$$

$$\lim_{\Delta r \to 0}\left\{ \frac{4\pi r^2 N_{Ar}|_r - 4\pi r^2 N_{Ar}|_{r+\Delta r}}{4\pi r^2 \Delta r} = 0 \right\}$$

$$\boxed{\frac{d}{dr}\left[r^2 N_{Ar}\right] = 0} \rightarrow r^2 N_{Ar} = \text{constant}$$

(b) (10 points) Using your result from part (a), plug in an expression for N_{Ar} to determine the governing differential equation for the molar rate of diffusion of liquid A away from the droplet, W_A. You can ***not*** assume we are dealing with a dilute solution, but the diffusion coefficient and the total concentration are constant.

$$W_A = 4\pi r^2 N_{Ar}, \qquad N_{Ar} = -c D_{AB} \frac{dy_A}{dr} + y_A (N_{Ar} + \underbrace{\cancel{N_{Br}}}_{0\ \text{- stagnant}})$$

$$\Rightarrow r^2 N_{Ar} = -\frac{c D_{AB}}{1-y_A} r^2 \frac{dy_A}{dr}$$

$$\Rightarrow \boxed{W_A = 4\pi r^2 N_{Ar} = -\frac{4\pi c D_{AB}}{1-y_A} r^2 \frac{dy_A}{dr}}$$

(c) (8 Points) Write down the boundary conditions you would use to arrive at an analytical expression for the molar rate of diffusion of liquid A away from the droplet. However, do **NOT** integrate the expression form part (b).

$$y_A = y_{A1} \quad @ \quad r = R_1$$

$$y_A = y_{A2} = 0 \quad @ \quad r = R_1 + \delta$$

(d) (7 Points) The following expression is the answer you would arrive at if you were to integrate the correct part (b) expression and apply the proper boundary conditions:

$$W_A\big|_{r=R_1} = 4\pi R_1^2 N_{Ar} = \frac{4\pi c D_{AB} R_1 (R_1+\delta)}{\delta} \ln\left(\frac{1}{1-y_{A1}}\right)$$

Using this expression immediately above, what is the expression for k_c, the convective mass transfer coefficient relating the concentration of A in the gas at the droplet surface to the concentration of A at $r = (R_1 + \delta)$?

$$\left.\begin{aligned} N_{Ar} &= k_c \Delta C_A = k_c (y_{A1} - 0) \cdot c \\ N_{Ar} &= \frac{c D_{AB} (R_1 + \delta)}{R_1 \delta} \ln\left(\frac{1}{1-y_{A}}\right) \end{aligned}\right\} \text{set equal}$$

$$\boxed{k_c = \frac{D_{AB}(R_1+\delta)}{R_1 \delta y_{A1}} \ln\left(\frac{1}{1-y_{A1}}\right)}$$

(e) (15 Points) Now you want to determine the amount of time it takes the radius of the droplet to decrease by half, and thus you do **NOT** assume that the radius of the liquid droplet, R_1, is constant. Set up a mole balance on the liquid droplet you would use to determine the amount of time it takes the radius of the droplet to decrease by half.

Do **NOT** solve; just set up the pertinent mole balance and **write down** the initial and final conditions you would use to solve it. State any assumptions or approximations made. Remember that you analyzed the gas phase only, not the liquid phase, in parts (a)-(d).

Pseudo-steady state approximation

balance on liquid drop

$$\cancel{In} - out + \cancel{gen} = Acc$$

In: 0 – no flux into drop

gen: 0 – no rxn.

$$out = W_A|_{r=R_1} \Rightarrow \text{from part (d)}$$

$$Acc = \frac{d}{dt}(C_A V)$$

$C_A \equiv$ concentration in liquid drop $=$ constant

$$\Rightarrow -W_A|_{r=R_1} = C_A \frac{dV}{dt}$$

$$V = \frac{4}{3}\pi R_1^3 \Rightarrow \frac{dV}{dt} = 4\pi R_1^2 \frac{dR_1}{dt}$$

mole balance final:

$$\boxed{-\frac{\cancel{4\pi} c D_{AB} \cancel{R_1}(R_1+\delta)}{\delta}\ln\left(\frac{1}{1-y_{A1}}\right) = \cancel{4\pi} R_1^{\cancel{2}}\frac{dR_1}{dt}}$$

- separate and integrate over t and R_1

$R_1 = R_{1,\text{initial}}$ @ $t=0$

$R_1 = R_{1,\text{final}} = \frac{R_{1,\text{initial}}}{2}$ @ $t = t_{\text{final}}$

Midterm #1 Solns/prob 2.

2.

(a) mol balance gives,

$$A_\perp N_A|_z - A_\perp N_A|_{z+\Delta z} = \frac{\partial(C_A A_\perp \Delta z)}{\partial t} = \emptyset \text{ at st. st.}$$

dividing by volume $A_\perp \Delta z$, in $\lim_{\Delta z \to \emptyset}$, get $\frac{\partial N_A}{\partial z} = 0$.

or $N_A = [\text{constant}]$. Because dilute, from def of flux, $N_A = -D_{AB}\frac{dC_A}{dz} = [\text{constant}]$ from mol balance. Integrating,

$$N_A \int_0^\delta dz = -D_{AB} \int_{C_A=\emptyset}^{C_A=C_{A\infty}} dC_A \quad \text{or} \quad \delta = -\frac{D_{AB}\, C_{A\infty}}{N_A}.$$

Now $N_A = -(\text{rxn rate/area})$, so $\delta = \dfrac{5\times10^{-9}\frac{m^2}{s} \cdot 2\ mol/m^3}{10^{-5}\ mols/2m^2}$

$= \boxed{2\ mm.}$

(b) mol balance gives,

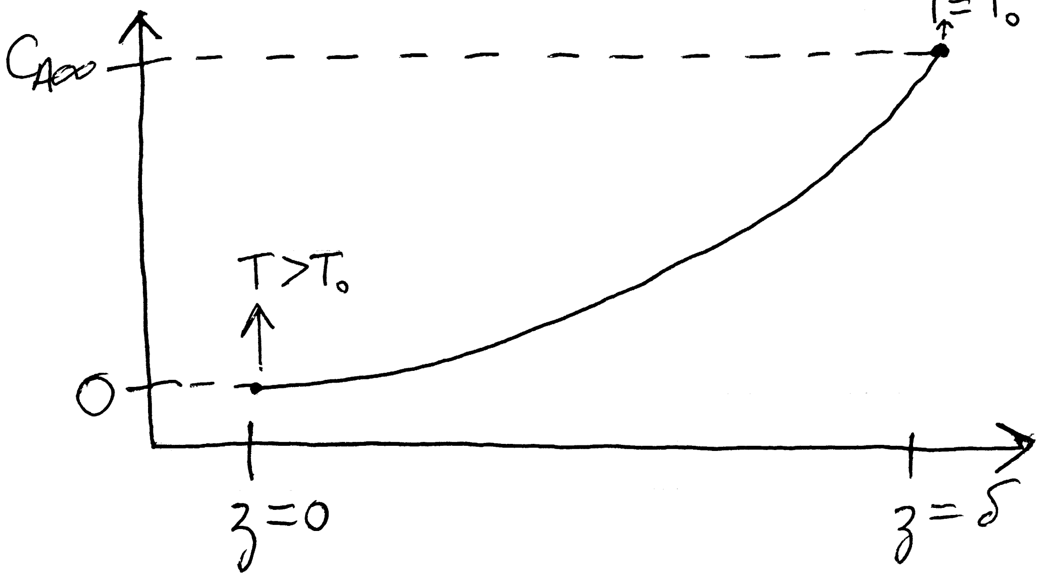
$$A_\perp N_A|_z - A_\perp N_A|_{z+\Delta z} = \frac{\partial(C_A A_\perp \Delta z)}{\partial t} = \emptyset \text{ at st. st.}$$

dividing by volume $A_\perp \Delta z$, in $\lim_{\Delta z \to 0}$, $\frac{\partial N_A}{\partial z} = 0$.

Now because dilute, $N_A = -D_{AC0}\left(\frac{T}{T_0}\right)\frac{dC_A}{dz}$.

so, $\boxed{\frac{\partial}{\partial z}\left(-D_{AC0}\left(\frac{T}{T_0}\right)\frac{dC_A}{dz}\right) = 0. \quad \text{BC \#1: } C_A = 0 \text{ @ } z=0 \quad \text{BC \#2: } C_A = C_{A\infty} \text{ @ } z=\delta.}$

(c)



<u>Lower</u>. Because mol balance requires N_A (total flux) to be constant along z direction and because $N_A \times \frac{T_0}{T} \propto \frac{dC_A}{dz}$, as z increases (T decreases), $\frac{dC_A}{dz}$ must increase.

part (b).