#1. From declassified pictures of an atomic blast, Geoffrey Taylor was able to predict the energy, E, of the atomic bomb solely from measurements of the radii of the blast, r, at a particular time, t, after the blast and knowing the ambient air density, ρ, and pressure, P.

a) Use the Buckingham Pi theorem with E, ρ, and t as core variables to determine relevant dimensionless parameters and write the relation between them in functional form.

b) Because the energy is very large, for small times after the explosion the dimensionless parameter containing P is nearly zero and the functional dependence on this dimensionless parameter can be considered constant. Knowing this and using your result from part a), you want to determine the energy for a different bomb. What parameters would you either have to know or measure if you wanted to predict the energy of this bomb?

Solution

a)

	r	t	P	ρ	E
M	0	0	1	1	1
L	1	0	-1	-3	2
T	0	1	-2	0	-2

5-3 = 2 dimensionless groups

$$\pi_1 = rt^a\rho^b E^c = L[T]^a\left[\frac{M}{L^3}\right]^b\left[\frac{ML^2}{T^2}\right]^c \Rightarrow a=-2/5,\ b=1/5,\ c=-1/5 \Rightarrow \pi_1 = rt^{-2/5}\rho^{1/5}E^{-1/5}$$

$$\pi_2 = Pt^d\rho^e E^f = \frac{M}{LT^2}[T]^d\left[\frac{M}{L^3}\right]^e\left[\frac{ML^2}{T^2}\right]^f \Rightarrow d=6/5,\ e=-3/5,\ f=-2/5 \Rightarrow \pi_2 = Pt^{6/5}\rho^{-3/5}E^{-2/5}$$

$$\pi_1 = rt^{-2/5}\rho^{1/5}E^{-1/5} = f(\pi_2) = f(Pt^{6/5}\rho^{-3/5}E^{-2/5})$$

b) The functional form becomes: $\pi_1 = rt^{-2/5}\rho^{1/5}E^{-1/5} = f(\pi_2) = constant$, so you can determine the energy/yield at a particular time after the explosion of a different bomb if you know the air density, time, and radius at that particular time .

#2 Solution:

This is very similar to the Chevron dust release problem from lecture, and the gravity settling chamber section of Denn.

a) The time for the particles to settle is simply the distance they must fall (5.2 m) divided by the terminal velocity of the spherical particle.

The terminal velocity V_p appears in both the drag coefficient C_D and the Re, so V_p may be found iteratively (assume Re<1, use Stokes' law to calculate C_D, calculate V_p from C_D, check Re with the V_p calculated, etc.) or this can be done graphically, by finding a quantity that is independent of V_p.

I calculated

$$C_D \mathrm{Re}^2 = \frac{4}{3}\frac{gD_p^3(\rho_p - \rho)\rho}{\eta^2} \qquad \text{(which is independent of } V_p\text{)}$$

with the data given, this is:

$$C_D \mathrm{Re}^2 = \frac{4}{3}\frac{gD_p^3(\rho_p - \rho)\rho}{\eta^2} = \frac{4}{3}\frac{(9.81\, m/s^2)(10^{-4}\, m)^3(2000 - 1.25\, kg/m^3)(1.25\, kg/m^3)}{(1.8x10^{-5}\ kg/m \cdot s)^2} = 101$$

so we could plot

$C_D = 101/\mathrm{Re}^2$ on our C_D vs Re plot. It would have a slope of -2 on the log-log plot, and go through a point where Re = 10 and C_D = 1.01. It would intersect the C_D vs Re plot at Re=3.43, C_D =8.59.

Rather than actually plotting on my C_D vs Re plot, I simply eye-balled where the curves would intersect, and realized it would be in the intermediate regime, where

$C_D \cong 18\mathrm{Re}^{-0.6}$. Substituting into our equation $C_D = 101/\mathrm{Re}^2$, we find Re=3.43, C_D =8.59.

From Re=3.43, we can calculate V_p, since $V_p = \dfrac{\mathrm{Re}\,\eta}{\rho D_p}$. The calculation yields

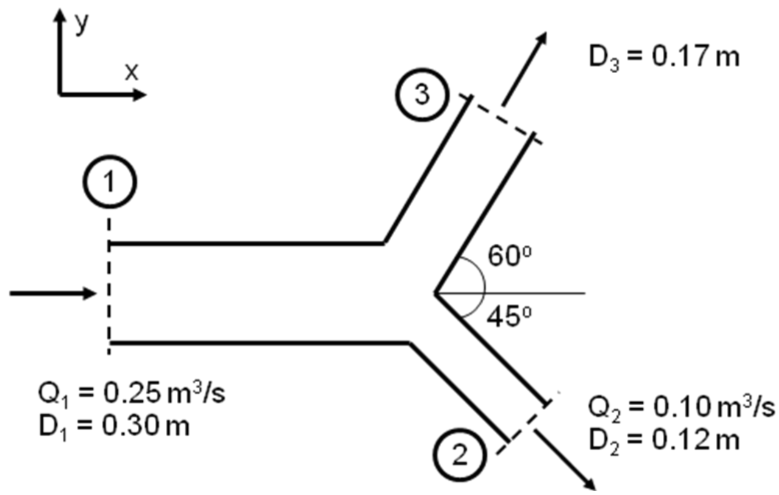
V_p = 0.494 m/s

The time to fall 5.2 m at this terminal velocity is simply 5.2 m/ V_p = 5.2 m/(0.494m/s) = 10.5 s

b) If the nearest town is 1600 m from the field, we want a wind speed such that the particles travel less than this distance in the time it takes them to fall to the ground, so

V_{wind} < (1600 m)/(10.5 s) = 152 m/s

#3.
Determine the force exterted by the fluid in the x and y direction on the apparatus shown. The apparatus is lying horizontally, that is, gravity acts in the -z direction. The gauge pressure at (1) is 7000 Pa, (2) is open to the atmosphere (atmospheric pressure is 1.01 x 10^5 Pa), and the gauge pressure at (3) is 4000 Pa. Assume turbulent flow of an incompressible fluid at steady state. The fluid has a density of 1000 kg/m^3. Draw arrows on the diagram below, indicating the direction of the forces that the fluid exerts on the surroundings.



Solution:
Start with the conservation of linear momentum. Since the system is at steady state and lying horizontally, neglect the time dependent and gravity term. Since the system is turbulent, set beta equal to unity:

$$\underline{F} = \rho < V >_1^2 \underline{A}_1 - \rho < V >_2^2 \underline{A}_2 - \rho < V >_3^2 \underline{A}_3 + P_1\underline{A}_1 - P_3\underline{A}_3$$

We need the velocity of the fluid. The velocity is related to flow rate:

$$v = \frac{Q}{A}$$

From conservation of mass, solve the flow rate at (3) of the incompressible fluid:

$$\rho Q_1 = \rho Q_2 + \rho Q_3 \Rightarrow Q_1 = Q_2 + Q_3$$

$$0.25 \text{ m}^3/\text{kg} = 0.10 \text{ m}^3/\text{kg} + Q_3 \Rightarrow Q_3 = 0.15 \text{ m}^3/\text{kg}$$

From the flow rate and diameters, calculate the fluid velocity at (1), (2), and (3):

$$v_1 = \frac{Q_1}{A_1} = \frac{Q_1}{(\pi/4)D_1^2} = \frac{0.25 \text{ m}^3/\text{kg}}{(\pi/4)(0.30 \text{ m})^2} = 3.54 \text{ m/s}$$

$$v_2 = \frac{Q_2}{A_2} = \frac{Q_2}{(\pi/4)D_2^2} = \frac{0.10\ \mathrm{m^3/kg}}{(\pi/4)(0.12\ \mathrm{m})^2} = 8.84\ \mathrm{m/s}$$

$$v_3 = \frac{Q_3}{A_3} = \frac{Q_3}{(\pi/4)D_3^2} = \frac{0.15\ \mathrm{m^3/kg}}{(\pi/4)(0.17\ \mathrm{m})^2} = 6.61\ \mathrm{m/s}$$

Now we decompose linear momentum equation into x and y components. Examine the x-component:

$$F_x = \rho v_1^2 A_1 - \rho v_2^2 A_2 \cos(\theta_2) - \rho v_3^2 A_3 \cos(\grave{e}_3) + P_1 A_{1,x} - P_3 A_{3,x}$$

$$F_x = \rho v_1 Q_1 - \rho v_2 Q_2 \cos(\theta_2) - \rho v_3 Q_3 \cos(\grave{e}_3) + P_1 A_{1,x} - P_3 A_{3,x}$$

$$+\rho v_1 Q_1 = \left(1000\,\tfrac{\mathrm{kg}}{\mathrm{m^3}}\right)\left(3.54\,\tfrac{\mathrm{m}}{\mathrm{s}}\right)\left(0.25\,\tfrac{\mathrm{m^3}}{\mathrm{s}}\right) = 884.2\ \mathrm{N}$$

$$-\rho v_2 Q_2 \cos(\theta_2) = -\left(1000\,\tfrac{\mathrm{kg}}{\mathrm{m^3}}\right)\left(8.84\,\tfrac{\mathrm{m}}{\mathrm{s}}\right)\left(0.10\,\tfrac{\mathrm{m^3}}{\mathrm{s}}\right)\cos(-45^\circ) = -625.1\ \mathrm{N}$$

$$-\rho v_3 Q_3 \cos(\grave{e}_3) = -\left(1000\,\tfrac{\mathrm{kg}}{\mathrm{m^3}}\right)\left(6.61\,\tfrac{\mathrm{m}}{\mathrm{s}}\right)\left(0.15\,\tfrac{\mathrm{m^3}}{\mathrm{s}}\right)\cos(60^\circ) = -495.6\ \mathrm{N}$$

$$+P_1 A_{1,x} = P_1(\pi/4)D_1^2 = (7000\ \mathrm{Pa})(\pi/4)(0.30\ \mathrm{m})^2 = 494.8\ \mathrm{N}$$

$$-P_3 A_{3,x} = -P_3(\pi/4)D_3^2\cos(\theta_3) = -(4000\ \mathrm{Pa})(\pi/4)(0.17\ \mathrm{m})^2\cos(60^\circ) = -45.4\ \mathrm{N}$$

$$F_x = 884.2\ \mathrm{N} - 625.1\ \mathrm{N} - 495.6\ \mathrm{N} + 494.8\ \mathrm{N} - 45.4\ \mathrm{N} = 213\ \mathrm{N}$$

Examine the y-component:

$$F_y = -\rho v_2^2 A_2 \sin(\theta_2) - \rho v_3^2 A_3 \sin(\grave{e}_3) - P_3 A_{3,y}$$

$$F_y = -\rho v_2 Q_2 \sin(\theta_2) - \rho v_3 Q_3 \sin(\grave{e}_3) - P_3 A_{3,y}$$

$$-\rho v_2 Q_2 \sin(\theta_2) = -\left(1000\,\tfrac{\mathrm{kg}}{\mathrm{m^3}}\right)\left(8.84\,\tfrac{\mathrm{m}}{\mathrm{s}}\right)\left(0.10\,\tfrac{\mathrm{m^3}}{\mathrm{s}}\right)\sin(-45^\circ) = 625.1\ \mathrm{N}$$

$$-\rho v_3 Q_3 \sin(\grave{e}_3) = -\left(1000\,\tfrac{\mathrm{kg}}{\mathrm{m^3}}\right)\left(6.61\,\tfrac{\mathrm{m}}{\mathrm{s}}\right)\left(0.15\,\tfrac{\mathrm{m^3}}{\mathrm{s}}\right)\sin(60^\circ) = -858.7\ \mathrm{N}$$

$$-P_3 A_{3,x} = -P_3(\pi/4)D_3^2\sin(\theta_3) = -(4000\ \mathrm{Pa})(\pi/4)(0.17\ \mathrm{m})^2\sin(60^\circ) = -78.6\ \mathrm{N}$$

$$F_y = 625.1\ \mathrm{N} - 858.7\ \mathrm{N} - 78.6\ \mathrm{N} = -312\ \mathrm{N}$$