ChE 150A - MidTerm Solutions

A.G.

1. Power = ?

$\langle v_1 \rangle = 0$ $\quad Q = 2\ m^3/min$

$\langle v_2 \rangle = 30\ m/s$ $\quad \varepsilon = 0.05\ \cancel{m}m$

$P_1 = P_2 = P_{atm}$ $\quad L = 100m$

$z_1 = z_2$ $\quad D = 10cm$

Bernoulli's Equation

$$\frac{\alpha_2}{2}\langle v_2\rangle^2 = \frac{\alpha_1}{2}\langle v_1\rangle^2 - \int_{P_1}^{P_2}\frac{dP}{\rho} + \delta W_s - l_v + g(z_1 - z_2)$$

$$\delta W_s = \frac{\alpha_2}{2}\langle v_2\rangle^2 + l_v \qquad \boxed{+5}$$

Since there is a nozzle, $A_2 \neq A_{pipe} \rightarrow \langle v_2\rangle \neq \langle v\rangle$

$$\langle v\rangle = \frac{Q}{A} = \frac{Q}{\pi D^2/4} = \frac{2m^3/min}{\pi(0.1m)^2(1/4)} = 4.24\ m/s \qquad \boxed{+3}$$

Check the Reynolds number

$$Re = \frac{\rho\langle v\rangle D}{\mu} = \frac{(10cm)(4.24\ m/s)(1\ g/cm^3)}{0.01\ g/cm\text{-}s} \qquad \boxed{+5}$$

$$= 4.24\times10^5 \rightarrow \text{turbulent flow} \quad \underline{\alpha_2 = 1}$$

$$f = (1.375\times10^{-3})\left[1 + \left(20000\frac{\varepsilon}{D} + \frac{10^6}{Re}\right)^{1/3}\right] \qquad \frac{\varepsilon}{D} = \frac{0.05\ cm}{10\ cm} = 5\times10^{-4}$$

$$= 4.55\times10^{-3} \qquad \cancel{\boxed{+5}}\ \boxed{+3}\ \text{(fixed)}$$

$$l_v = \frac{2\langle v\rangle^2 L}{D}\cdot f = \frac{2(4.24\ m/s)^2(100m)}{(0.1m)}(4.55\times10^{-3})$$

$$= 164.1\ m^2/s^2 \qquad \boxed{+3}$$

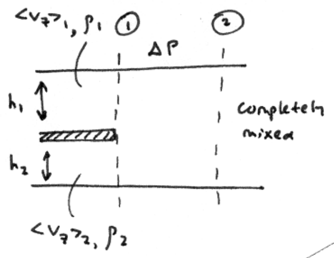
$$\delta W_s = \frac{1}{2}(30\ m/s)^2 + 164.1\ m^2/s^2 = 614.1\ m^2/s^2$$

$$\text{Power} = \underbrace{w\cdot\delta W_s}_{\boxed{+3}} = \rho Q\,\delta W_s = \left(\frac{1g}{cm^3}\right)\left(\frac{2m^3}{min}\right)\left(\frac{100cm}{m}\right)^3\left(\frac{min}{60s}\right)\left(614.1\frac{m^2}{s^2}\right)$$

$$P = 2.05\times10^4\ W \qquad \boxed{+3}$$

A.G.

2.)



Momentum Balance : $F=0$, neglect gravity

$$\rho_1 A_1 \beta_1 \langle v_1 \rangle^2 + P_1 A_1 = \rho_2 A_2 \beta_2 \langle v_2 \rangle^2 + P_2 A_2 \qquad \boxed{+5}$$

$A_1 = A_2$, $\rho_1 = \rho_2$

$\beta_2 = 1$, uniform velocity

Mass Balance

$$\rho_1 A_1 \langle v_1 \rangle = \rho_2 A_2 \langle v_2 \rangle$$

$$\langle v_1 \rangle = \langle v_2 \rangle \qquad \boxed{+3}$$

$\boxed{+5}$ β_1, β_2

Simplifying Momentum Balance : $P_2 - P_1 = \rho \beta_1 \langle v_1 \rangle^2 - \rho \langle v_1 \rangle^2$

$$\beta_1 = \frac{\langle v_1^2 \rangle}{\langle v_1 \rangle^2}$$

$$\boxed{P_2 - P_1 = \rho \langle v_1^2 \rangle - \rho \langle v_1 \rangle^2} \qquad \boxed{+2}$$

1

Need to determine $\langle v_1 \rangle$, average velocity at pt. 1

$$\langle v_1 \rangle = \frac{1}{(h_1+h_2)} \left[\int_0^{h_2} \langle v_{z2} \rangle \, dy + \int_{h_2}^{h_1+h_2} \langle v_{z1} \rangle \, dy \right]$$

$$\boxed{\langle v_1 \rangle = \frac{h_2 \langle v_{z2} \rangle + h_1 \langle v_{z1} \rangle}{h_1 + h_2}} \qquad \boxed{+5}$$

$$\left.\begin{aligned} \langle v_1 \rangle^2 &= \left[\frac{h_2 \langle v_{z2} \rangle + h_1 \langle v_{z1} \rangle}{h_1 + h_2} \right]^2 \\ \langle v_1^2 \rangle &= \left[\frac{h_2 \langle v_{z2}^2 \rangle + h_1 \langle v_{z1}^2 \rangle}{h_1 + h_2} \right] \end{aligned}\right\} \boxed{+5}$$

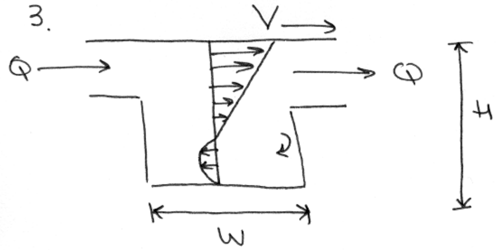
Have now defined all the things were after in 1.

Note: ρ is an average density over the cross-sectional area

Comment: There are alternate ways to solve this problem, I have just shown one such way.

A.G

3.



let the z direction be characterized by L

a. $v_x = ?$

Assume
- s.s. $\frac{\partial}{\partial t} \to 0$
- incompressible
- $v_y = v_z = 0$
- creeping flow
- ignore z-direction

any $\boxed{+1}$

Boundary Conditions

(i) $v_x = 0 \quad y = 0$ $\boxed{+1}$

(ii) $v_x = V \quad y = H$ $\boxed{+1}$

Continuity

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \implies \frac{\partial v_x}{\partial x} = 0 \quad \boxed{+2}$$

for check & writing continuity

$v_x(y)$ only

Navier-Stokes

$$x: \quad 0 = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 v_x}{\partial y^2}$$

$$y: \quad 0 = -\frac{\partial P}{\partial y}$$

$$z: \quad 0 = -\frac{\partial P}{\partial z}$$

$P(x)$ $\boxed{+3}$

$$+\frac{\partial P}{\partial x} = \mu \frac{\partial^2 v_x}{\partial y^2} = k \quad \boxed{+2}$$

Two functions each depending on a different variable, if they are equal, they must be equal to a constant

$$k = \frac{\Delta P}{W}$$

$$\implies \frac{1}{\mu} \frac{\Delta P}{W} = \frac{d^2 v_x}{dy^2}$$

$$v_x = \frac{1}{2\mu} \frac{\Delta P}{W} \cdot y^2 + C_1 y + C_2 \quad \boxed{+2}$$

A.G.

use B.C. to solve for C_1 & C_2

(i) $v_x = 0, y = 0 \rightarrow C_2 = 0$

(ii) $v_x = V, y = H \rightarrow C_1 = \frac{V}{H} - \frac{1}{2\mu}\frac{\Delta P}{W} \cdot H$

$$\boxed{v_x = \frac{1}{2\mu}\frac{\Delta P}{W}(y^2 - Hy) + \frac{V}{H}\cdot y}$$ $\boxed{+3}$

b. $\frac{\Delta P}{W} = ?$

$Q = \langle v_x \rangle \cdot A$ $\qquad \langle v_x \rangle = \frac{1}{A}\int v_x \, dA$ $\qquad A = H \cdot L$

$= \langle v_x \rangle H \cdot L$ $\qquad \langle v_x \rangle = \frac{1}{H}\int_0^H v_x \, dy$ $\boxed{+2}$ $\qquad dA = H \cdot dy$

$\frac{Q}{L} = \langle v_x \rangle \cdot H$

$$= \int_0^H \left[\frac{1}{2\mu}\frac{\Delta P}{W}(y^2 - Hy) + \frac{V}{H}y\right] dy \cdot \frac{1}{H}$$

$$= \left[\frac{1}{2\mu}\frac{\Delta P}{W}\left(\frac{1}{3}y^3 - \frac{1}{2}Hy^2\right)\Big|_0^H + \frac{1}{2}\frac{V}{H}y^2\Big|_0^H\right]\frac{1}{H}$$

$$= \left[\frac{H^3}{2\mu}\frac{\Delta P}{W}\left(\frac{1}{3} - \frac{1}{2}\right) + \frac{1}{2}VH\right]\frac{1}{H}$$

$$= \left[-\frac{\Delta P H^3}{12\mu W} + \frac{1}{2}VH\right]\frac{1}{H} = -\frac{\Delta P \cdot H^2}{12\mu W} + \frac{1}{2}V$$

$$\frac{Q}{L} = \frac{-\Delta P H^3}{12\mu W} + \frac{1}{2}VH$$

$$\boxed{\frac{\Delta P}{W} = \frac{-12\mu}{H^3}\left(\frac{Q}{L} - \frac{1}{2}VH\right)}$$ $\boxed{+3}$

This reduces to the solution for $Q = 0$ seen in HW #7

$$\frac{\Delta P}{W}(Q = 0) = \frac{6\mu V}{H^2}$$

c. $F|_{y=H} = ?$

$$F = \tau_{yx}\Big|_{y=H} \cdot A = \tau_{yx}\Big|_{y=H} \cdot W \cdot L \quad \boxed{+2}$$

$$\tau_{yx} = \mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

$$= \mu \frac{\partial v_x}{\partial y} \quad \boxed{+1}$$

$$F = \mu L W \frac{\partial v_x}{\partial y}\Big|_{y=H}$$

$$= \mu L W \left[\frac{1}{\mu}\frac{\Delta P}{W}\left(y - \frac{H}{2}\right) + \frac{V}{H}\right]\Big|_{y=H}$$

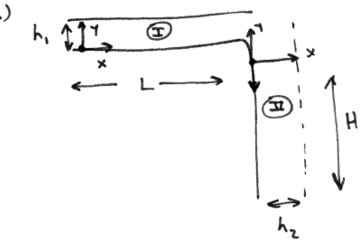
$$= \mu L W \left[\frac{H}{2\mu}\frac{\Delta P}{W} + \frac{V}{H}\right] \qquad \frac{\Delta P}{W} = -\frac{12\mu}{H^3}\left(\frac{Q}{L} - \frac{1}{2}VH\right) \text{ from b.}$$

$$= \frac{\mu L W V}{H} + \frac{L W H}{2}\left(-\frac{12\mu}{H^3}\left(\frac{Q}{L} - \frac{1}{2}VH\right)\right)$$

$$= \frac{\mu L W V}{H} + \frac{3\mu L W V}{H} - \frac{6\mu Q W L}{H^2}$$

$$\boxed{F = \frac{4\mu V W L}{H} - \frac{6\mu Q W L}{H^2}} \quad \boxed{+2}$$

4.)



(A.) I $v_x(y)$ $\boxed{+1}$

N.S. $$0 = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 v_x}{\partial y^2} \quad \boxed{+2}$$

B.C. $v_x = 0$ at $y = 0$ $\boxed{+1}$

$v_x = 0$ at $y = h_1$ $\boxed{+1}$

$$\rightarrow v_x = \frac{1}{\mu}\frac{\partial P}{\partial x}\left[\frac{y^2}{2} - \frac{h_1 y}{2}\right] \quad \boxed{+2}$$

II $v_y(x)$ $\boxed{+1}$

N.S. $$0 = -\frac{\partial P}{\partial y} + \mu \frac{\partial^2 v_y}{\partial x^2} \quad \boxed{+2}$$

B.C. $v_y = 0$ at $x = 0$ $\boxed{+1}$

$\frac{\partial v_y}{\partial x} = 0$ at $x = h_2$ $\boxed{+1}$

$$\rightarrow v_y = \frac{1}{\mu}\frac{\partial P}{\partial y}\left[\frac{x^2}{2} - h_2 x\right] \quad \boxed{+2}$$

(B.) $$q_I = \frac{Q}{W} = \int_0^{h_1} v_x \, dy = \int_0^{h_1} \frac{1}{\mu}\frac{\partial P}{\partial x}\left[\frac{y^2}{2} - \frac{h_1 y}{2}\right] dy \quad \boxed{+2}$$

$$\rightarrow q_I = \frac{-h_1^3}{12\mu}\frac{\partial P}{\partial x} \quad \boxed{+2}$$

$$q_{II} = \int_0^{h_2} v_y \, dx = \int_0^{h_2} \frac{1}{\mu}\frac{\partial P}{\partial y}\left[\frac{x^2}{2} - h_2 x\right] dx$$

$$\rightarrow q_{II} = \frac{-h_2^3}{3\mu}\frac{\partial P}{\partial y} \quad \boxed{+2}$$

(C.) $q_{II} = q_I$, conservation of mass $\boxed{+3}$

So, $$\frac{q_{II}}{q_I} = 1 = \frac{\frac{-h_2^3}{3\mu}\frac{\partial P}{\partial y}}{\frac{-h_1^3}{12\mu}\frac{\partial P}{\partial x}} \rightarrow h_2 = \left[4 h_1^3 \frac{\partial P}{\partial x}\cdot\frac{\partial y}{\partial P}\right]^{1/3} \quad \boxed{+2}$$

Comment: It is ok if you wrote that: $P(x) = p + \rho g x$

$P(y) = p + \rho g y$

In region I $\frac{\partial P}{\partial x} = \frac{\Delta P}{L} = \frac{P_{atm} - P_o}{L}$

In region II $\frac{\partial P}{\partial y} = \rho g H$