UNIVERSITY OF CALIFORNIA, BERKELEY College of Engineering

CE130-I: The First Mid-term Examination

Problem 1.

Derive the equilibrium equation for a two-dimensional infinitesimal element in vertical (Y) direction. Note that the thickness of the element (Z-direction) is taken as 1 (unit length), and X, Y are the body forces with the unit (force per unit volume). (20 points)

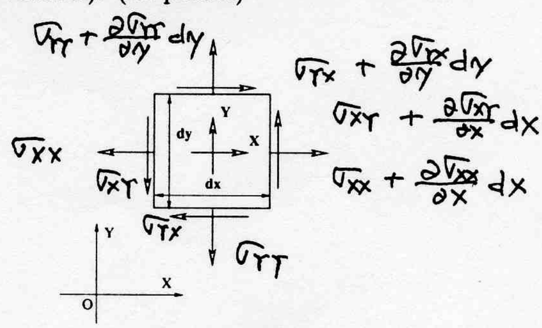


Figure 1: A 2D infinitesimal element

Problem 2

Given a 2D displacement fields as

$$u_x = 2x^2 + xy + 3 \quad (1)$$
$$u_y = 2x^2 + 1 \quad (2)$$

(1) Find $\epsilon_{xx}, \epsilon_{yy}, \epsilon_{xy}$ at the point $(1,1)$; Assume that this is also a plane stress state, i.e. $\sigma_{zz} = 0.0$. (2) Find σ_{xx}, σ_{yy} and σ_{xy} at the point $(1,1)$ assuming Young's modulus $E = 100MP_a$ and Poission's ratio $\nu = 0.25$ (Shear modulus $G = E/(2(1+\nu))$). (20 points)

Hints:

$$\epsilon_{xy} = \frac{1}{2}\Big(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\Big) \quad (3)$$
$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \quad (4)$$
$$\epsilon_{yy} = \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{zz}}{E} \quad (5)$$

Problem 3

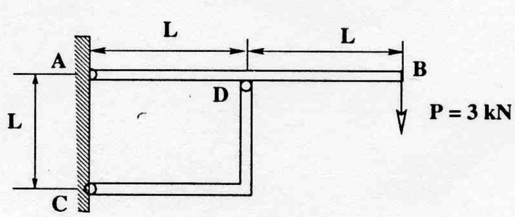


Figure 2: A two-bar bracket system

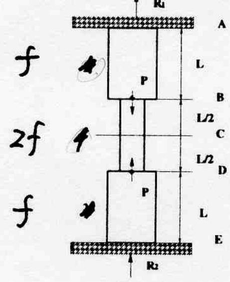


Figure 3: A three-bar statically indeterminant system

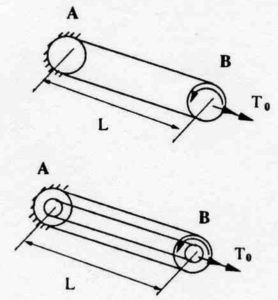


Figure 4: Torsion of two shaft systems

A wall bracket is constructed as shown in the Figure 3. All joints may be considered pin connected. Find the shear stress in the bolt A. Note that bolt A is 20 mm in diameter and it acts in double shear. (20 points)

Problem 4

Consider a three elastic bar system (statically indeterminant) shown in the Figure 4. Find the reaction forces R_1 and R_2.

Problem 5

Two cylinder shafts, one solid cylinder with radius R and one hollow cylinder with outer radius $r_o = 1.2R$ and inner radius $r_i = 0.6R$, are subjected by the same torques, T_0, as indicated in Figure 5. Calculate and compare the maximum shear stress inside each shaft.

Hint: the polar moment of inertia is defined as

$$J = \int_A r^2 dA = \int_{R_i}^{R_o} r^3 dr d\theta \tag{6}$$

where R_i is the inner radius of the shaft and R_o is the outer radius of the shaft.

Torsion formula : $\tau_{max} = \frac{T \cdot R_o}{J}$

The Solution of First Midterm Exam

Problem 1

$\Sigma F_Y = 0,$

$$\left(\sigma_{YY} + \frac{\partial \sigma_{YY}}{\partial Y} dY\right)(dx \cdot 1) - \sigma_{YY}(dx \cdot 1)$$

$$+ \left(\sigma_{XY} + \frac{\partial \sigma_{XY}}{\partial X} dx\right)(dY \cdot 1) - \sigma_{XY}(dY \cdot 1)$$

$$+ Y \cdot (dx \cdot dY \cdot 1) = 0$$

$$\Rightarrow \quad \frac{\partial \sigma_{YY}}{\partial Y} dx\,dY + \frac{\partial \sigma_{XY}}{\partial X} dx\,dY + Y\,dx\,dY = 0$$

$$\Rightarrow \quad \boxed{\frac{\partial \sigma_{XY}}{\partial X} + \frac{\partial \sigma_{YY}}{\partial Y} + Y = 0}$$

Problem 2

$u_x = 2x^2 + xy + 3, \quad u_y = 2x^2 + 1$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = 4x + y, \quad \varepsilon_{YY} = \frac{\partial u_y}{\partial y} = 0, \quad \varepsilon_{xY} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_Y}{\partial x}\right)$$

$$= \frac{1}{2}(x + 4x) = \frac{5x}{2}$$

At point (1,1),

$$\varepsilon_{xx} = 5, \quad \varepsilon_{YY} = 0, \quad \varepsilon_{xY} = \frac{5}{2}$$

$$\underline{\varepsilon} = \begin{bmatrix} 5 & 5/2 \\ 5/2 & 0 \end{bmatrix}$$

By Generalized Hooke's law and $\sigma_{zz} = 0$

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{yy}}{E} = 5 \qquad (1)$$

$$\varepsilon_{yy} = -\nu \frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} = 0 \qquad (2)$$

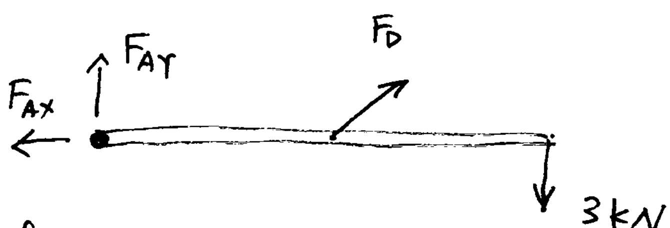
From (2) $\Rightarrow$ $\sigma_{yy} = \nu \sigma_{xx}$

From (1) $\Rightarrow$ $\sigma_{xx}(1-\nu^2) = 5E$ $\Rightarrow$ $\sigma_{xx} = \dfrac{5 \times 100 \times 10^6}{1 - 0.25^2} = 533 \times 10^6$ Pa

and $\sigma_{yy} = \nu \sigma_{xx} = 0.25 \times 533$ MPa $= 133$ MPa

$$\sigma_{xy} = 2G\varepsilon_{xy} = \frac{E}{(1+\nu)} \cdot \frac{5}{2} = \frac{5 \times 10^8}{2(1.25)} = 200 \text{ MPa}$$

Problem 3

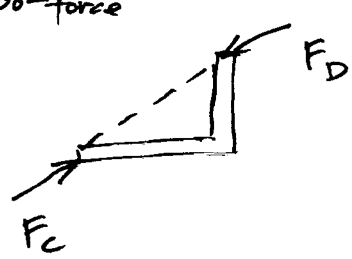


$\Sigma M_A = 0$ ↺+

$-3kN\,(2L) + F_D \frac{\sqrt{2}}{2} L = 0$

$F_D = 6\sqrt{2}$ kN

CD is a two-force member



Then

$$\Sigma F_x = 0$$

$$-F_{Ax} + F_D \frac{\sqrt{2}}{2} = 0 \qquad F_{Ax} = \frac{\sqrt{2}}{2} \cdot 6\sqrt{2} = 6 \text{ kN}$$

$$\Sigma F_Y = 0$$

$$F_{AY} - 3 \text{ kN} + 6\sqrt{2} \cdot \frac{1}{\sqrt{2}} \text{ kN} = 0$$

$$F_{AY} = -3 \text{ kN} \quad (\downarrow)$$

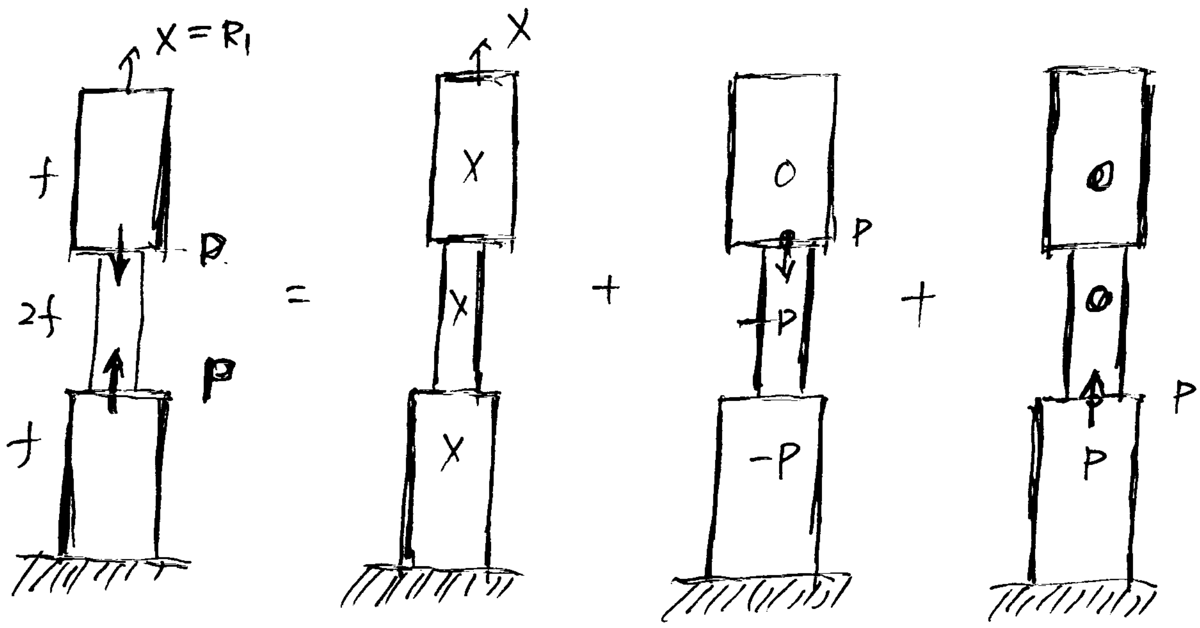
$$F_A = \sqrt{F_{Ax}^2 + F_{AY}^2} = \sqrt{36 + 9} = \sqrt{45}$$

$$\tau = \frac{F_A}{2A} = \frac{\sqrt{45} \times 10^3}{2 \times \pi (10 \times 10^{-3})^2}$$

$$= \frac{\sqrt{45} \times 10}{2\pi} \text{ MPa}$$

$$\cong 10.68 \text{ MPa}$$

Problem 4



$\Delta^1 = fX + 2fX + fX = 4fX$

$\Delta^2 = 2f(-P) + f(-P) = -3fP$

$\Delta^3 = fP$

$\Delta = \Delta^1 + \Delta^2 + \Delta^3 = 4fX - 3fP + fP = 4fX - 2fP = 0$

$X = \frac{P}{2}$; $\Rightarrow R_1 = \frac{P}{2}$

$\Sigma F_x = 0$ $\quad R_1 - P + P + R_2 = 0$ $\quad R_2 = -P_1 = -\frac{P}{2}$

Problem 5.

$$J_{solid} = \int_0^{2\pi}\int_0^{R} r^3 dr\, d\theta = \frac{\pi R^4}{2}$$

$$J_{hollow} = \int_0^{2\pi}\int_{0.6R}^{1.2R} r^3 dr\, d\theta = \frac{\pi}{2}\left((1.2R)^4 - (0.6R^4)\right)$$

$$= \frac{\pi R^4}{2}\left(1.2^4 - 0.6^4\right) = \frac{\pi R^4}{2}(1.2)^4\left(1-\left(\tfrac{1}{2}\right)^4\right) = \frac{\pi R^4}{2}(1.2)^4\frac{15}{16}$$

$$= 1.944\,\frac{\pi R^4}{2} = 1.944\,J_s$$

$$\tau_{solid} = \frac{T^0 \cdot R}{J_s}, \qquad \tau_{hollow} = \frac{T^0\,(1.2R)}{1.944 J_s}$$

$$\frac{\tau_{solid}}{\tau_{hollow}} = \frac{T^0 R / J_s}{1.2\,T^0 R / 1.944 J_s} = \frac{1.944}{1.2} = 1.62$$

$$\tau_{solid} > \tau_{hollow}$$