ChE 150A Fluid Mechanics and Heat Transfer
Mid-Term Examination
Closed Book

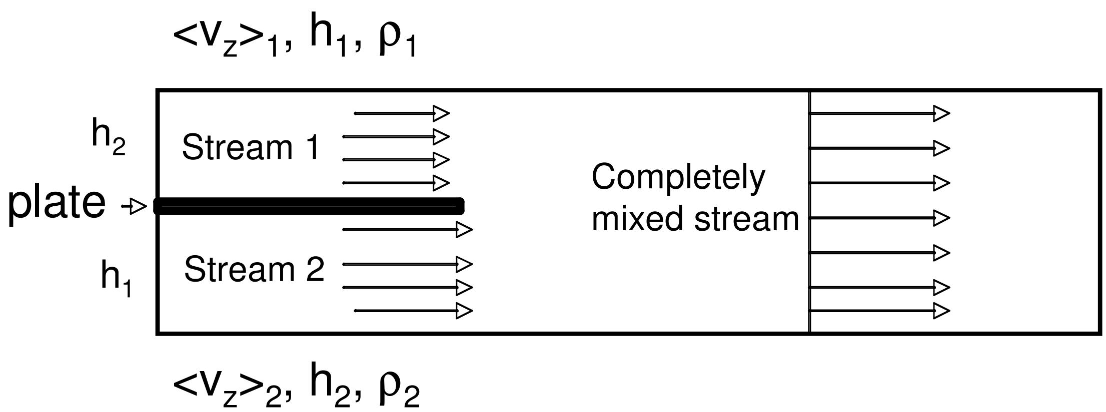
9:00-10:00am, 23rd March, 2007
All questions have equal value

Q1. A fire truck is sucking water from a river and delivering it through a long hose to a nozzle, from which it issues at a velocity of 30 meters/sec. The total flow rate is 2 cubic meters/min. The hose has a diameter of 10 cm, and has an internal roughness of ε=0.05 mm. The total length of the hose, corrected for all the valves, fittings and other losses, is 100 meters. What is the power required for the fire truck's pump?

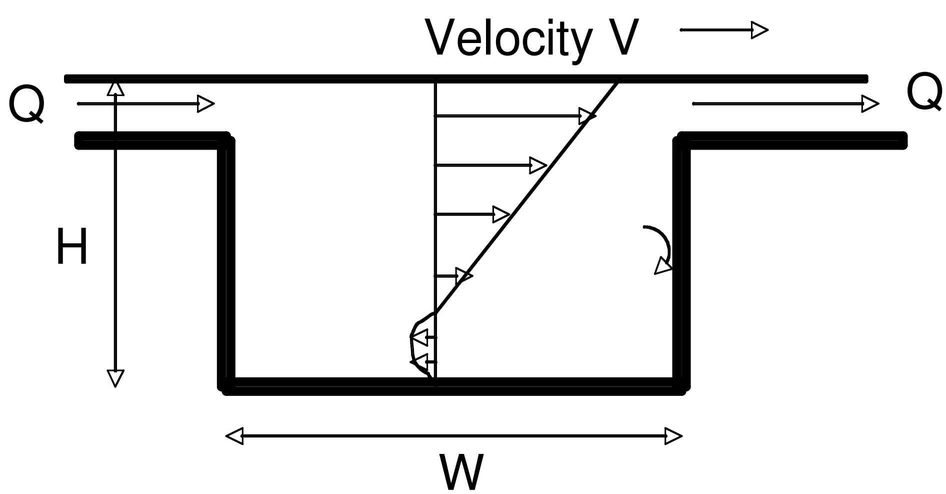
In the turbulent regime, the friction factor f can be determined from the equation below, where Re is the Reynold's number, ε/D is the dimensionless roughness and D is the diameter of the pipe.

$$f = 0.001375\left[1+\left(20,000\frac{\varepsilon}{D}+\frac{10^6}{\mathrm{Re}}\right)^{1/3}\right]$$

Q2. Two miscible turbulent streams of densities ρ_1 and ρ_2 are flowing in a wide, rectangular duct separated by a thin plate. The heights of the two streams are h_1 and h_2. Neglecting viscous effects, use the momentum balance to calculate the pressure change in the mixing region in terms of $\langle v_z\rangle_1$,$\langle v_z\rangle_2$, h_1 and h_2. Assume the pressure is uniform across the channel and the velocity profiles of the unmixed and completely mixed streams are flat.

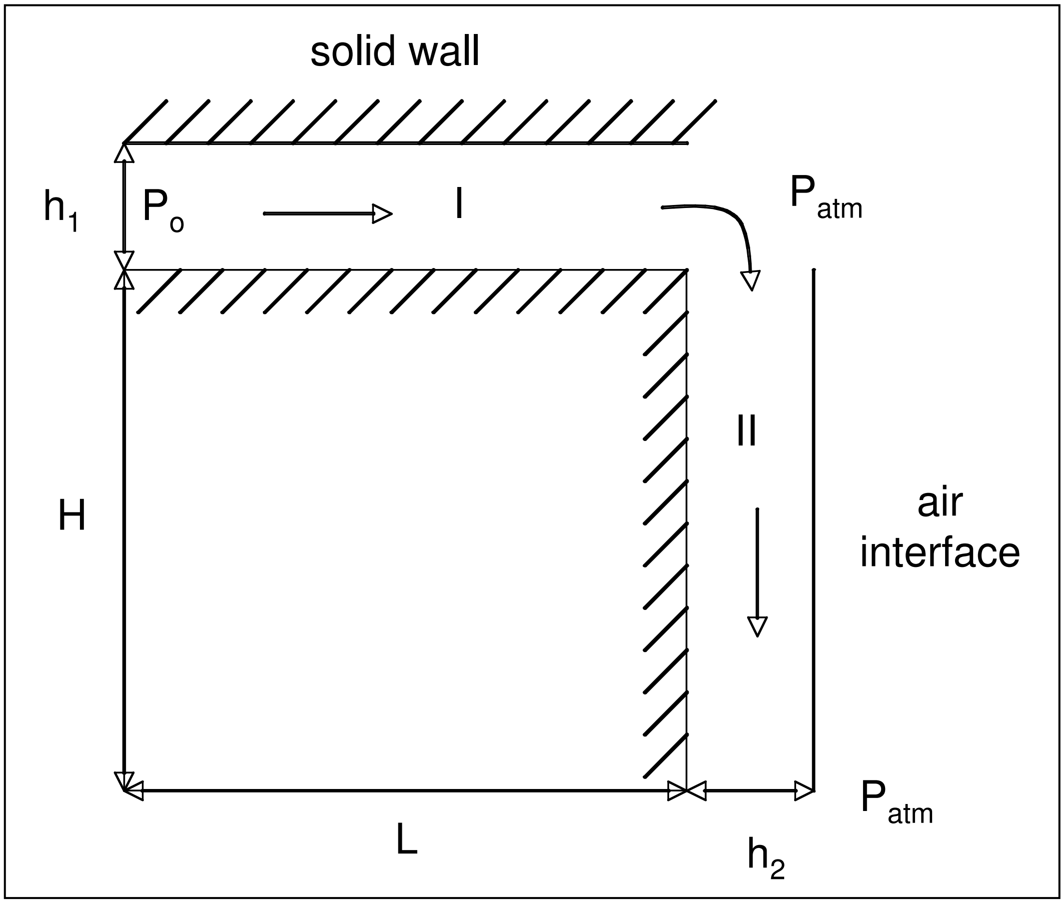


Q3. A simplified model of a screw extruder used in polymer processing is depicted below. The top plate moves with a velocity V and molten polymer enters and leaves the system at a flow rate Q. The flow profile is indicated on the figure, and consists of a pressure driven Poiseuille flow and a recirculating Couette flow in the opposite direction. The dimensions of the extruder channel are indicated on the figure.
(a) Determine the velocity profile in the center of the channel (neglect the recirculation of the fluid at the ends)
(b) What is the pressure change across the length (W) of the channel?
(c) What is the force required to move the upper plate at the velocity V?



Q4. Consider the following problem: there is pressure-driven flow between two flat plates in region I, which then pours over a ledge into region II, where it is in contact with air. Assume that the flow is creeping in region II and touches the wall. Ignore entrance and exit effects in both regions.

a. Determine the velocity profile in each region. Define your coordinate axes and clearly state your assumptions and boundary conditions.
b. Determine the volumetric flowrate per unit width in each region. Assume the width, W, in both regions is equal.
c. Determine h_2 in terms of h_1 and other parameters.



solid wall
h1
Po
I
Patm
II
H
air
interface
L
h2
Patm

NOTES:

The Navier-Stokes equations for a constant viscosity, constant density fluid in rectangular coordinates are (note P is the equivalent pressure):

$$\rho(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}) = -\frac{\partial P}{\partial x} + \mu(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2})$$

$$\rho(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}) = -\frac{\partial P}{\partial y} + \mu(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2})$$

$$\rho(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}) = -\frac{\partial P}{\partial z} + \mu(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2})$$

The continuity equation is

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

The stress constitutive equation for a Newtonian fluid with constant density in rectangular coordinates is

$$\tau_{xy} = \tau_{yx} = \mu(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x})$$

$$\tau_{zy} = \tau_{yz} = \mu(\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z})$$

$$\tau_{xz} = \tau_{zx} = \mu(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z})$$

The density of water is 1 gm/ml and its viscosity is 1 cp (0.01 gm/cm-sec) at ambient conditions.