UNIVERSITY OF CALIFORNIA
Industrial Engineering and Operations Research

Midterm Exam
Spring 01

LINEAR PROGRAMMING – IEOR 162

Instructor Juan Carlos Muñoz

Attempt all four questions. Show all your work. If you feel you need more space use the back of the preceding page. As a guideline, the following allocation of your time (in minutes) is suggested.

Problem 1 (Graphical Solution and Sensitivity Analysis)	10-15
Problem 2 (Interpreting Computer Package Output)	5-10
Problem 3 (Modeling)	15-20
Problem 4 (Minimax functions)	5
Total	35-55

Notice that the maximum score is 110 points, however 100 points will be considered a perfect performance.

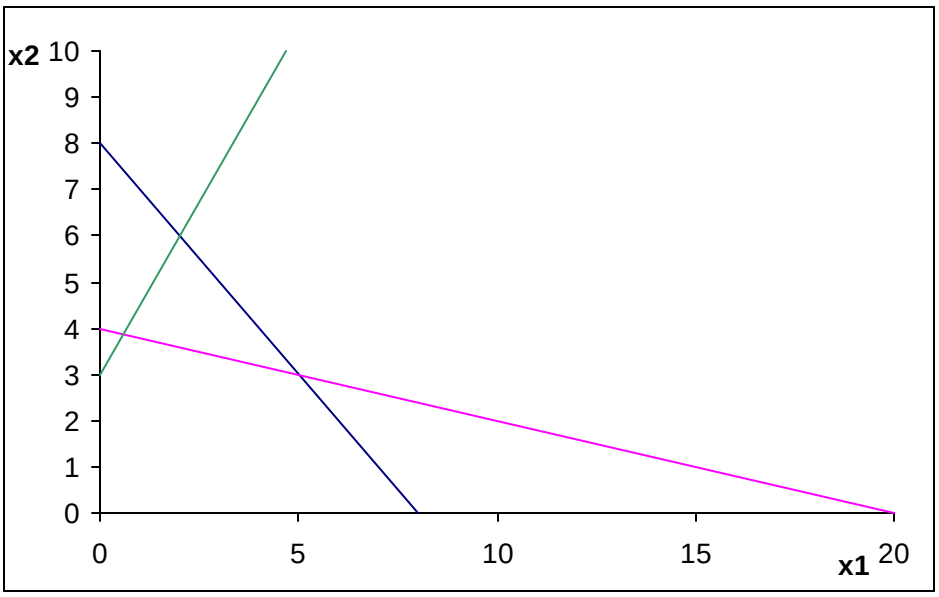
Problem	1	2	3	4	Total
Score	30	30	30	20	100
Your score					

Name:_____

1) Graphical Solution and Sensitivity Analysis (30 points)

The constraints of the following LP are graphed.

$$\begin{aligned} &\textit{Min} && z = 3x_1 + 5x_2 \\ &\textit{s.t.} \\ & && x_1 + 5x_2 \geq 20 && \text{(A)} \\ & && -3x_1 + 2x_2 \geq 6 && \text{(B)} \\ & && x_1 + x_2 \geq 8 && \text{(C)} \\ & && x_1 \geq 0 \\ & && x_2 \geq 0 \end{aligned}$$



a) (2 points) Label the constraints with the letters given above

b) (2 points) Shade in the feasible region

c) (2 points) Draw an isoprofit line and show the direction of improvement

d) (4 points) Determine the optimal solution

$(x_1, x_2, e_1, s_2, e_3) =$

$z =$

e) (6 points) How large could the coefficient of x2 in the objective function be without changing the optimal solution?

f) (7 points) Determine the shadow price and the range of optimality of the right hand side of constraint A that doesn't change the set of binding constraints

Shadow price of constraint A is: _____

The maximum value of the right hand side of constraint A is:_____

The minimum value of the right hand side of constraint A is:_____

g) (7 points) Determine the shadow price and the range of optimality of the right hand side of constraint B that doesn't change the set of binding constraints

Shadow price of constraint B is: _____

The maximum value of the right hand side of constraint B is:_____

The minimum value of the right hand side of constraint B is:_____

2) Interpreting Computer Package Output (30 points)

Carco manufactures cars and trucks. Each car contributes $300 to profit, and each truck contributes $400. The resources required to manufacture a car and a truck are shown in the following table.

	Days on Type 1 Machine	**Days on Type 2 Machine**	**Tons of Steel**
Car	0.8	0.6	2
Truck	1	0.7	3

Each day, Carco can rent up to 98 type 1 machines at a cost of $50 per machine. At present, the company has 73 type 2 machines and 260 tons of steel available. Marketing considerations dictate that at least 88 cars and at least 26 trucks be produced. Let x_1=number of cars produced daily; x_2= number of trucks produced daily; and m_1=type 1 machines rented daily.

To maximize profit, Carco should solve the LP showed in Fig. 7 for Problem 2 (section 5.2 of the textbook, page 223). Use the LINDO output to answer the following questions:

a) (6 points) What is the optimal production and renting plan and profit?
Cars:
Trucks:
Type 1 Machines:
Profit:

b) (6 points) If each truck contributed $380 to profit, what would be the new optimal solution and profit?
Cars:
Trucks:
Type 1 Machines:
Profit:

c) (6 points) If Carco were required to produce at least 86 cars, what would Carco's profit become?

Profit:

d) (6 points) Carco has the opportunity to purchase some extra tons of steel. What is the maximum unit price Carco should be willing to pay? What is the maximum amount that Carco should buy?

e) (6 points) An external firm is requesting Carco to rent one of the machine types 1 for $250. Should Carco rent the machine? Give a clear explanation.

3) Modeling (30 points)

The SAVE-IT COMPANY operates a reclamation center that collects three types of solid waste materials and treats them so that they can be amalgamated into a salable product (treating and amalgamating are separate processes). Two different grades of this product can be made depending upon the mix of materials used. Although there is some flexibility in the mix for each grade, quality standards may specify the minimum or maximum amount allowed for the proportion of a material in the product grade (this proportion is the weight of the material expressed as a percentage of the total weight for the product grade). These specifications are given in the following table, along with the amalgamation cost and the selling price for each grade:

Grade	Specification	Amalgamation Cost per pound ($)	Selling Price per Pound ($)
A	Material 1: at most 30% of total Material 2: at least 40% of total Material 3: exactly 20% of total	3.00	8.50
B	Material 1: Not more than 70% of total	2.00	5.50

The reclamation center collects its solid waste materials from regular sources and so is normally able to maintain a steady rate for treating them. The following table gives the quantities available for collection and treatment each week, as well as the cost of treatment, for each type of material.

Material	Pounds per week available	Treatment cost per pound ($)
1	3,000	3.00
2	2,000	6.00
3	1,000	4.00

The Save-It Co. is solely owned by Green Earth, an organization devoted to dealing with environmental issues, so Save-It's profits are used to help support Green Earth's activities. Green Earth has raised contributions and grants, amounting to $30,000 per week, to be used exclusively to cover the entire treatment cost for the solid waste materials. The board of directors of Green Earth has instructed the management of Save-It to divide this money among the materials in such a way that at least half of the amount available of each material is actually collected and treated.

Management wants to determine the amount of each product grade to produce and the exact mix of materials to be used for each grade so as to maximize the total weekly profit.

4) Minimax functions (20 points)

Transform the following mathematical program into an equivalent linear program (if possible):

$$
\begin{aligned}
&\mathit{Min}\left(\max\left\{x_1 - 2, x_1 + 3x_2 - 6\right\}\right)\\
&\mathit{s.t.}\quad \left|x_1 - x_2\right| \le 3\\
&\qquad\quad x_2 \le 4
\end{aligned}
$$