Operations Research II, IEOR161
University of California, Berkeley
Midterm Exam II, 2009

Name:
Student ID:

1. [**15**] An airline reservation system has 2 indistinguishable computers, of which only one can be used at any given time. A computer in use may break down on any given day with probability p (a computer not in use cannot break down). There is a single repair facility that takes 2 days to restore a computer to normal. The facility is such that it only begins repairs in the beginning of the day and it can only deal with one computer at a time. Form a Markov chain by taking as states the pairs (x, y) where x is the number of machines in operating condition at the beginning of a day and y is 1 if a day's worth of repair has been expended on a machine not yet operational and 0 otherwise.

   (a) What are the 4 possible values for the state (x, y)?

   (b) What is the transition matrix/diagram?

   (c) Write down the equations for the stationary distribution (do not solve them).

2. [**15**] What is the stationary distribution of a 3 state Markov chain (with states $\{1, 2, 3\}$) and transition matrix

$$P = \begin{bmatrix} 0.4 & 0.2 & 0.4 \\ 0 & 0.4 & 0.6 \\ 0.8 & 0.2 & 0 \end{bmatrix}.$$

3. [**15**] Consider a Poisson process with rate λ. Let S_1 be the time of the first arrival, S_2 the time of the second arrival, S_3 the time of the third arrival (etc). Calculate

$$E\Big\{ \sum_{j=1}^{N(T)} S_j \Big\}$$

   Explain your working.

4. [**15**] Men arrive according to a Poisson process of rate $\lambda = 3$ (per hour) while women arrive according to a Poisson process of rate $\mu = 5$ (per hour). The arrival processes for men and women are independent. Given that 8 people (men or women) arrived between 12pm and 1pm, what is the probability they all arrived between 12:15pm and 12:45pm?