**IEOR 161 Operations Research II**
**University of California, Berkeley**
**Spring 2008**

**Midterm1 Suggested Solution**

1.

(a). Let X be the time it takes for the bridge to collapse and Y be the time it takes for Indiana Jones to cross the bridge. $\lambda_1 = \frac{1}{2}$, $\lambda_2$=2 be the corresponding rates.

$$\begin{aligned} Pr\{X > 1+Y\} &= \int_{y=0}^{\infty} Pr\{X > 1+y\} Pr\{Y = y\} dy \\ &= \int_{y=0}^{\infty} e^{-\lambda_1(1+y)} \lambda_2 e^{-\lambda_2 y} dy \\ &= \lambda_2 e^{-\lambda_1} \int_{y=0}^{\infty} e^{-(\lambda_1+\lambda_2)y} dy \\ &= \frac{\lambda_2 e^{-\lambda_1}}{\lambda_1+\lambda_2} = \frac{2}{2.5} e^{-0.5} = \frac{4}{5} e^{-0.5} \end{aligned}$$

Another way to do this problem:

$$Pr\{X > 1+Y\} = Pr\{X > 1+Y | X > 1\} Pr\{X > 1\} = \frac{\lambda_2}{\lambda_1+\lambda_2} e^{-\lambda_1} = \frac{4}{5} e^{-0.5}$$

(b). Indiana Jones falls into the ravine if the collapse occurs after 1 hour and before he finishes crossing the bridge.

$$\begin{aligned} Pr\{1 < X < 1+Y\} &= Pr\{X < 1+Y\} - Pr\{X < 1\} \\ &= (1 - Pr\{X > 1+Y\}) - (1 - Pr\{X > 1\}) \\ &= Pr\{X > 1\} - Pr\{X > 1+Y\} \\ &= e^{-0.5} - \frac{2e^{-0.5}}{2.5} = \frac{1}{5} e^{-0.5} \end{aligned}$$

2.

(a). Let X be the time until the next time Gwenaelle makes the deposit, and Y be the time until the parents make the next check. $\lambda$=0.5 and $\mu$=1.

$$Pr\{X < Y\} = \frac{\lambda}{\lambda+\mu} = \frac{0.5}{1.5} = \frac{1}{3}$$

(b). Let T be the time till they need to make a diaper change.

$$\begin{aligned} E[T] &= E[T|X < Y] Pr\{X < Y\} + E[T|X > Y] Pr\{X > Y\} \\ &= \left(\frac{1}{\lambda+\mu} + \frac{1}{\mu}\right) \frac{\lambda}{\lambda+\mu} + \left(\frac{1}{\lambda+\mu} + E[T]\right) \frac{\mu}{\lambda+\mu} \\ E[T] &= \frac{1}{\lambda} + \frac{1}{\mu} = 3 \end{aligned}$$

(c). 1) Assuming that the "deposit" and "checking" processes are independent.
2) Assuming that the time between babies making deposits in her diaper is exponential.
3) Assuming that the time it takes to change the diaper is instantaneous.

3. Let the rainy season be the time interval (0,1) and let N(1) be the number of storms in the season.

$$\begin{aligned}
Pr\{N(1) \leq 2|\Lambda\} &= e^{-\Lambda}(1 + \Lambda + \frac{\Lambda^2}{2}) \\
Pr\{N(1) \leq 2\} &= \int_{\lambda=0}^{\infty} Pr\{N(1) \leq 2|\Lambda = \lambda\} Pr\{\Lambda = \lambda\} d\lambda \\
&= \int_{\lambda=0}^{\infty} e^{-\lambda}(1 + \lambda + \frac{\lambda^2}{2}) \frac{1}{3} e^{-\frac{\lambda}{3}} d\lambda \\
&= \frac{1}{3} \int_{\lambda=0}^{\infty} e^{-\frac{4}{3}\lambda}(1 + \lambda + \frac{\lambda^2}{2}) d\lambda
\end{aligned}$$