Solutions to IEOR 130 Midterm Examination
Spring 2004
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1. (a) Countable parameter, so use C-chart

(b) $Cpk = (USL - \mu)/3\sigma = (USL - \mu)/3*SQRT(\mu)$

$UCL = 60 = \mu + 3SQRT(\mu)$

$60 - \mu = 3SQRT(\mu)$

$3600 - 120\mu + \mu^2 = 9\mu$

$3600 - 129\mu + \mu^2 = 0$

Using quadratic formula, we find $\mu = 40.83$

Hence $Cpk = (50 - 40.83)/3SQRT(40.83) = 0.478$

(c) Yield = Prob { $X < USL$ } = Prob { $Z < (USL - \mu)/\sigma$ } = $\Phi(3Cpk) = \Phi(1.435) = 0.924$

(d) For Prob = 0.95, we want 3Cpk = 1.65, i.e., Cpk = 0.55

2. (a) $MY = Y_R + 3*SQRT[\, Y_R(1 - Y_R)/GD \,]$

$(0.85 - Y_R)^2 = (9/GD)\,[\, Y_R - Y_R^2 \,]$

$0.7225 - 1.7*Y_R + Y_R^2 = 0.009*Y_R - 0.009*Y_R^2$

$0.7225 - 1.709*Y_R + 1.009*Y_R^2 = 0$

Using quadratic formula, we find $Y_R = 0.8129$

(b) $Y_R^{new} = \exp(A\Delta D)*Y_R = \exp(0.5*0.05)*Y_R = 1.025*Y_R = 0.8335$

$MY^{new} = Y_R^{new} + 3*SQRT[\, Y_R^{new}(1 - Y_R^{new})/GD \,] = 0.869$

3. (a) 6 lots is 1.5 furnace loads. U = (1.5)(8)/24 = 0.5

(b) 0.5 = U = (expected time consumed by furnace cycles between O ring replacements) / (expected total time between O ring replacements)

Hence expected total time between O ring replacements = 2*(expected time consumed by furnace cycles)

(c) Minimize (O ring replacement time + expected time consumed by bad furnace run) / (expected total time between O ring replacements)

or Minimize (O ring replacement time + expected time consumed by bad furnace run) / [2* (expected time consumed by furnace cycles)]

or Minimize G(t) = (2 + 8 Σ k=1 to t p_k) / 2*[8*t (1 – Σ k=1 to t p_k) + 8*Σ k=1 to p_k k*p_k]

where p_k's are given in the problem.

(d)

t	pt	Σpk	1-Σpk	Σkpk	t(1-Σpk)	Numerator	Denominator	G(t)
1	0.10	0.10	0.90	0.10	0.90	2.80	16.0	0.175
2	0.15	0.25	0.75	0.40	1.50	4.00	30.4	0.1315
3	0.20	0.45	0.55	1.00	1.65	5.60	42.4	0.132
4	0.30	0.75	0.25	2.20	1.00	8.00	51.2	0.156
5	0.25	1.00	0.00	3.25	0.00	10.00	52.0	0.192

It's best to plan O ring replacements after every 2 furnace runs.