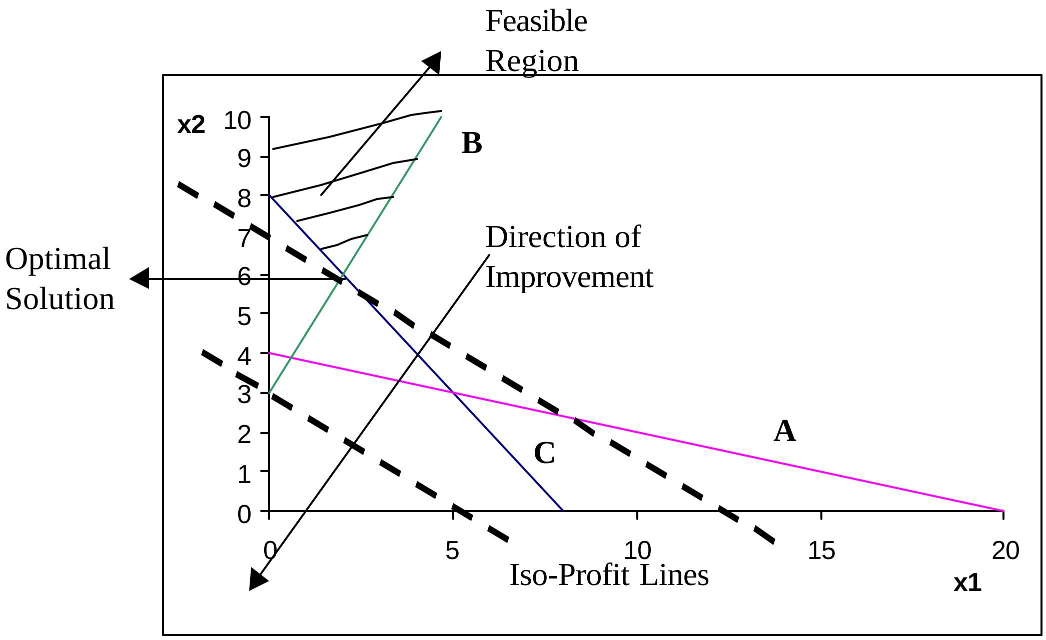
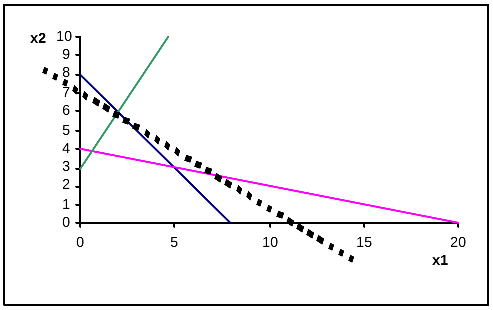
Mid-Term 1 Solution

1. (a), (b), (c)



(d)



Excess on non-binding Constraint

$$(X_1, X_2, e_1, e_2, e_3) = (2,6,12,0,0)$$

$$Z = 3\times 2 + 5\times 6 = 36$$

(e)

The optimal solution changes when the slope of the iso-profit line is steeper than constraint C.

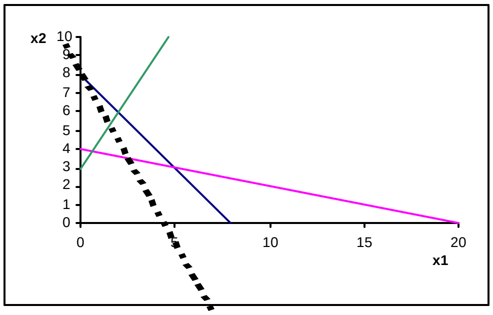


Figure shows a Case where the slope is steeper than constraint C and the solution changes.

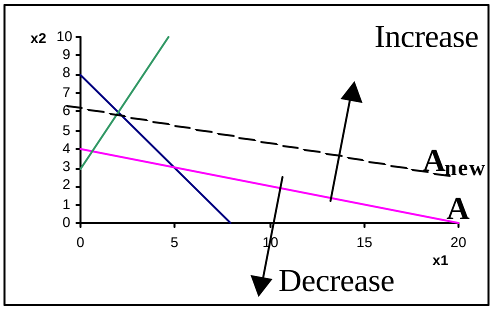
Slope of Constraint C = -1

Let C_1 = Coefficient for X_1
C_2 = Coefficient for X_2

C_1 Remaining Constant, we can see that with $C_2 \geq 3$ -- solution still remains optimal. $\Rightarrow 3 \leq C_2 \leq \infty$

(f)

Since A is not a binding constraint – the Shadow Price is 0.



To find the maximum RHS for A, we need to know the RHS value for which A will be a Binding Constraint – which will change the basis and hence the solution.
Hence we need to find the equation for the line **A**$_{new.}$

The value of the RHS of **A**$_{new}$ must be such that **A**$_{new}$ passes through the optimal point (2, 6).

Using this Information, we can find the equation of line **A**$_{new}$ $\Rightarrow x_1 + 5x_2 = 32$
$\Rightarrow$ **The maximum RHS value for A is 32**.

As in the figure above, the **Lower Limit for the RHS is** $-\infty$, as the solution/basis is not affected on decreasing the RHS for **A**.

(g)
The Shadow Price of a constraint is by how much the value of the objective function is increased (for a maximization problem) or decreased (for a minimization problem), if we change the RHS of that constraint by 1 unit, keeping in mind that the set of binding constraints remains unchanged.
Hence, let us change the RHS of constraint B by l

Solving the inequalities: -

$$X_1 + X_2 = 8$$
$$-3X_1 + 2X_2 = 6 + l$$

We get,

$$X_2 = 6 + \frac{l}{5}$$
$$X_1 = 2 - \frac{l}{5}$$

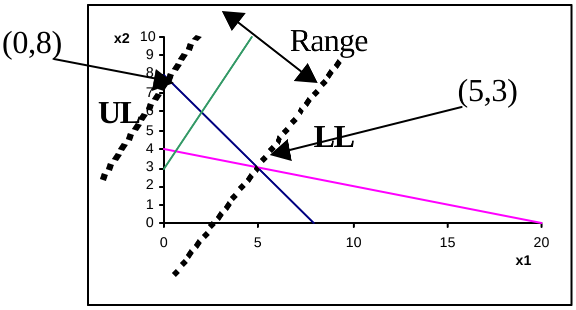
Hence,

$$Z = 36 + \frac{2l}{5}$$

As this is a Minimization Problem, the Objective Function decreases by $\frac{2}{5}$

Hence Shadow Price for B is $\frac{-2}{5}$

Figure shows the Range in which the RHS of Constraint B can vary, without changing the optimal basis.



Equation of the Upper Limit line – UL is $-3x_1 + 2x_2 = -3\times(0) + 2\times(8) = 16$
Equation of the Lower Limit Line – LL is $-3x_1 + 2x_2 = -3\times(5) + 2\times(3) = -9$

Hence, **Max RHS value for B is 16, and Min RHS value is –9**.

2

(a)

Cars	88
Trucks	27.6
Type 1 Machines	98
Profit	32540

(b)

Cars	88
Trucks	27.6
Type 1 Machines	98
Profit	32540 – 20(27.6) = 31988

(c)

Profit = Old Profit + (Shadow Price for the Constraint)(Change in Cars Produced)
= 32540 + (-20)(86-88)
= 32580

(d)

They **should not pay** anything, and they **should not buy any more steel**, since they do not use all the available steel (*the constraint is not binding*).

(e)

Carco **should not rent** the Machine.
The Shadow Price of Machine 1 Constraint is $ 350, which means the profit will go down by $ 350 if the Machine is rented out, whereas in return earnings will be only $ 250 – hence, there will be a net loss of $ 100.

3.

Decision variables

Amount of a Particular Material used to form a product of a particular Grade.
Material Types – 1, 2, 3
Product Grades – A, B

$$
\begin{aligned}
X_{1A} &= 1 \rightarrow A \\
X_{2A} &= 2 \rightarrow A \\
X_{3A} &= 3 \rightarrow A \\
X_{1B} &= 1 \rightarrow B \\
X_{2B} &= 2 \rightarrow B \\
X_{3B} &= 3 \rightarrow B
\end{aligned}
$$

Objective Function: Maximize Weekly Profit

$$Z = \overbrace{\{8.5(X_{1A} + X_{2A} + X_{3A}) + 5.5(X_{1B} + X_{2B} + X_{3B})\}}^{\text{Selling Price}} - \underbrace{\{3(X_{1A} + X_{2A} + X_{3A}) + 2(X_{1B} + X_{2B} + X_{3B})\}}_{\text{Treatment Cost}}$$

Subject to:

Proportion Constraints of materials in a Product Grade

$$
\left.\begin{array}{l}
\dfrac{X_{1A}}{X_{1A} + X_{2A} + X_{3A}} \le 0.3 \\[2ex]
\dfrac{X_{2A}}{X_{1A} + X_{2A} + X_{3A}} \ge 0.4 \\[2ex]
\dfrac{X_{3A}}{X_{1A} + X_{2A} + X_{3A}} = 0.2 \\[4ex]
\dfrac{X_{1B}}{X_{1B} + X_{2B} + X_{3B}} \le 0.7
\end{array}\right\}
\xrightarrow{\text{Or}}
\left\{\begin{array}{l}
0.7X_{1A} - 0.3X_{2A} - 0.3X_{3A} \le 0 \\
-0.4X_{1A} + 0.6X_{2A} - 0.4X_{3A} \ge 0 \\
-0.2X_{1A} - 0.2X_{2A} + 0.8X_{3A} = 0 \\
0.3X_{1B} - 0.7X_{2B} - 0.7X_{3B} \le 0
\end{array}\right.
$$

$$
\left.\begin{array}{l}
X_{1A} + X_{1B} \le 3000 \\
X_{2A} + X_{2B} \le 2000 \\
X_{3A} + X_{3B} \le 1000
\end{array}\right\} \text{Material Availability Constraints}
$$

$$
\left.\begin{array}{l}
X_{1A} + X_{1B} \ge 1500 \\
X_{2A} + X_{2B} \ge 1000 \\
X_{3A} + X_{3B} \ge 500
\end{array}\right\}
$$

"At least half of the amount available for each material is collected and treated" – i.e. lower Bound Constraint

$$3(X_{1A} + X_{1B}) + 6(X_{2A} + X_{2B}) + 4(X_{3A} + X_{3B}) \le 30000$$

Green Earth's contribution and Grants - constraint

All variables ≥ 0

4.

Let

$$a = Max\{X_1 - 2, X_1 + 3X_2 - 6\}$$

$$m = |X_1 - X_2|$$

Min $Z = a$

Such that

$$a \geq X_1 - 2$$

$$a \geq X_1 + 3X_2 - 6$$

$$X_1 - X_2 \leq 3$$

$$X_2 - X_1 \leq 3$$

$$X_2 \leq 4$$