1. **[5+5+5]** Paris has 4 good friends, Nikki, Britney, Lindsay, and Stavros, and enjoys spending time with each of them. To add variety to her life, Paris decides not to meet the same person on consecutive nights. Instead, she adopts a policy where if she goes out with one friend on a given evening, then she chooses equally between the other three for the next. For example, if she parties with Lindsay on Sunday night, then she parties with equal probability with Britney, Stavros, or Nikki on Monday.

   (a) Formulate Paris's social life as a discrete time finite state Markov chain, clearly identifying the states and transition probability matrix.

   (b) What proportion of time (in steady state) does Paris spend with each of her friends?

   (c) Suppose Paris spends Sunday night at Hugh's post-Oscar party with Nikki. What is the expected number of days before she meets with Stavros?

2. **[5+5]** Green men arrive at the launching pad according to a Poisson process with rate $\mu = 2$ while red men arrive according to a Poisson process with rate $\lambda = 3$. The space ship leaves when there are 3 men (of either color) at the pad.

   (a) What is the expected time required to fill the space ship?

   (b) What is the probability that the crew are all green?

3. **[10]** New organisms are born at random times distributed according to a Poisson process with rate λ. Each organism is alive for a random amount of time having an exponential distribution with rate μ before it dies. Find the probability that none of the organisms alive at time t were born before time s, where $s < t$.