IEOR 172
Mid-Term Exam I
October 14, 2008

Answer both questions.

Q1. Suppose that you continually collect coupons and that there are m different types. Suppose also that each time a new coupon is obtained it is a type i coupon with probability p_i, $i = 1, 2, \ldots, m$ ($\sum_{i=1}^{m} p_i = 1$). What is the probability that the n-th coupon you collect is different from the first $n-1$ coupons (that is, the n-th coupon is a new type)?

Q2.

Part 1: Let X be the number of trials needed to get the first success. Suppose the probability of success in a trial is p and $\mathtt{E}[X] = 5$. Find the second moment and the variance of X (state the assumptions you are making for this analysis). If Y is the number of failed trials before the first success, find the first and second moments and the variance of X

Part 2: Let X and Y be two random variables with the joint probability mass functions $f_{X,Y}$ given by

$$f_{X,Y}(1,1) = .06; f_{X,Y}(1,2) = .09; f_{X,Y}(1,3) = .12; f_{X,Y}(1,4) = .03;$$

$$f_{X,Y}(2,1) = .08; f_{X,Y}(2,2) = .12; f_{X,Y}(2,3) = .16; f_{X,Y}(2,4) = .04;$$

$$f_{X,Y}(3,1) = .06; f_{X,Y}(4,2) = .09; f_{X,Y}(5,3) = .12; f_{X,Y}(6,4) = .03.$$

Find the probability mass functions of X and Y. Are these two random variables statistically independent? Let $Z = X + Y$. Find the mean, second moment and variance of Z.