Operations Research II, IEOR161
University of California, Berkeley
Midterm Exam I, 2009

1. **[10 + 10 + 10]** Men arrive at a store as a Poisson process of rate $\lambda = 2$ (per hour) while women arrive as a Poisson process of rate $\mu = 3$ (per hour). Arrival processes for men and women are independent.

   (a) What is the probability that the first arrival is a man?
   Hint: How does this relate to the minimum of 2 exponential rv's?

   (b) What is the probability that the first 3 arrivals are men?

   (c) If men spend on average \$10 per visit while women spend \$20 per visit, what is the expected amount earned by the store from the first customer?

2. **[10+10]** There are 80 students in taking an exam. The exam lasts for 1 hour. The completion time for each student is independent and exponential with **mean** 0.75 (i.e. 3/4 of an hour).

   (a) What is the probability that 1/4 of the class leaves early?

   (b) What is the expected number of students who will finish early?

3. **[10 + 10]** Consider a small grocery store which has 2 checkout counters. There is a server at each counter. The service time of each server is an exponential random variable, where the rate of server 1 is μ_1 while that of server 2 is μ_2. Customers arrive according to a Poisson process with rate λ. Customers arriving at the counter and finding both servers busy wait in a queue. The customer at the head of the line goes into service when a server becomes free. Assume that service times and arrivals are independent.

   (a) Suppose both servers are busy. What is the probability that there are exactly two people in the queue when one of the servers finishes service?

   (b) Suppose that both servers are currently idle. What is the expected time until both servers are busy?