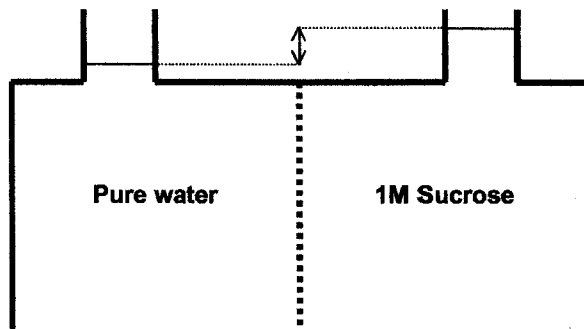


NAME:

2. The containers pictured below are connected by a semipermeable membrane, which allows water to pass freely but blocks transport of sucrose. The tube diameters are 1 cm, their area is $\pi(0.5)^2 \text{ cm}^2$, the height difference between liquid levels on each side is h (marked with the arrow), the density of both liquids is $\sim 1 \text{ g/ml}$, and the total volume of the containers is large.



- a. (5 pts) What is the pressure difference across the semipermeable membrane at equilibrium?

$$\pi = MRT$$

$$= \left(1 \frac{\text{mol}}{\text{L}}\right) (0.0821 \frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}) (298 \text{ K}) = 24.5 \text{ atm}$$

- b. (5 pts) What height difference (h) between the liquid levels will result at equilibrium?

$$P = \frac{F}{A} = \pi = \frac{mg}{A} = \frac{\rho Ahg}{A} \Rightarrow \pi = \rho gh$$

$$h = \frac{\pi}{\rho g} = \frac{24.5 \text{ atm}}{(1 \text{ g/mL})(9.81 \text{ m/s}^2)} = \frac{24.8 \times 10^6 \text{ Pa} \leftarrow \text{J/m}^2}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 253 \text{ m}$$

- c. (15 pts) What is the maximum amount of work (per liter) that can be extracted from the process of pure water and 1M sucrose, initially at equal pressure, coming to equilibrium in the configuration pictured above.

$$W = \pi \bar{V} \Rightarrow \frac{W}{\bar{V}} = \pi = 24.5 \text{ atm} \cdot \frac{101.3 \text{ J/L}}{1 \text{ atm}} = 2481 \text{ J/L}$$

or solve with chemical potentials:

$$\mu_{L(H_2O)} = \mu_{H_2O}^0 + n_{H_2O} RT \ln(X_{H_2O})$$

$$\mu_{R(H_2O)} = \mu_{H_2O}^0 + n_{H_2O} RT \ln(X_{H_2O}) + \pi \bar{V}$$

$\mu_L = \mu_R$ @ equilibrium, so:

$$-n_{H_2O} RT \ln X_{H_2O,R} = +\pi \bar{V} = W \leftarrow \text{work comes from the entropy of mixing!}$$

$$\frac{W}{\bar{V}} = \pi = \frac{-n_{H_2O,R} RT \ln X_{H_2O,R}}{\bar{V}} = \frac{(-55.6 \text{ mol})(0.0821)(298 \text{ K}) \ln\left(\frac{55.6}{56.6}\right)}{1 \text{ L}}$$

\uparrow
24.5 atm from (a)

$$\approx 24.3 \text{ atm}$$

approximation

NAME:

3. You may find the following approximations useful:

$$\phi_0(\sigma) = \frac{\sigma L_D}{\epsilon_w}$$

$$L_D \approx \frac{10}{\sqrt{I}} \text{ nm}$$

ϕ_0 = surface potential

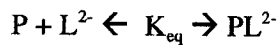
L_D = Debye length

ϵ_w = dielectric constant of the water (this is a positive constant; you don't need its numerical value)

I = ionic strength (e.g. [NaCl]) in units of mM

σ = surface charge density in units of charge per area

A membrane protein (in a membrane) binds a negatively charged, soluble ligand (L^{2-}):



$$\Delta G = \Delta G^0 + RT \ln Q - 2F\phi_0$$

a. (10 pts) You measure K_{eq} for this reaction in negatively charged membranes (e.g. $\sigma < 0$ so $\phi_0 < 0$) how will changing the ionic strength affect K_{eq} ? Justify your answer quantitatively with appropriate equations

$$\Delta G = 0 \Rightarrow K_{eq} = e^{-\frac{\Delta G^0}{RT}} \cdot e^{\frac{2F\phi_0}{RT}}$$

$$I \uparrow \Rightarrow L_D \downarrow \text{ (negatively) } \Rightarrow K_{eq} \uparrow$$

Problem continues on next page ...

NAME:

3. (continued) Suppose you discover a mutant ligand that binds with the same ΔG° but exists as a heterodimer (LX), which is net neutral: $P + (LX) \rightarrow P(LX)$

b. (7.5 pts) Write an expression for ΔG of this reaction.

$$\Delta G = \Delta G^\circ + RT \ln Q$$

c. (7.5 pts) Under what conditions, if any, would you expect the equilibrium binding constant of the mutant ligand, $K_{eq}(\text{mutant})$, to be the same as K_{eq} for the regular ligand? (Explain with appropriate equations)

$\phi_0 = 0$ full credit

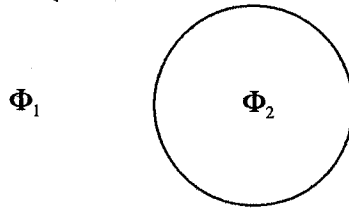
$$K_{eq}^{\text{mutant}} \stackrel{?}{=} K_{eq}$$

||

$$e^{-\frac{\Delta G^\circ}{RT}} = e^{-\frac{\Delta G^\circ}{RT}} \cdot e^{\frac{2\phi F}{RT}}$$

NAME:

4. Consider a lipid vesicle, which consists of a spherical lipid membrane enclosing a volume of solution as pictured below. The electrostatic potential outside the vesicle, Φ_1 , differs from the electrostatic potential inside, Φ_2 .



- a. (5 pts) Suppose an ion channel in this vesicle's membrane allows Na^+ ions to pass freely. Using the Boltzmann equation, write an expression that relates the concentration of Na^+ ions inside and outside the vesicle at equilibrium.

$$[\text{Na}^+]_{\text{out}} = [\text{Na}^+]_{\text{in}} e^{\frac{zFV}{RT}} \quad V = \phi_2 - \phi_1$$

- b. (10 pts) Derive the expression you gave in part (a) from chemical potentials. Hint: all you need to do is write down the correct chemical potentials, apply the appropriate equilibrium condition, and with a couple algebraic steps you will have it.

$$\mu_{\text{in}} = \mu^\circ + RT \ln [\text{Na}^+]_{\text{in}} + zF\phi_2$$

$$\mu_{\text{out}} = \mu^\circ + RT \ln [\text{Na}^+]_{\text{out}} + zF\phi_1$$

$$\Delta\mu = RT \ln \frac{[\text{Na}^+]_{\text{in}}}{[\text{Na}^+]_{\text{out}}} + zF(\phi_2 - \phi_1) = 0 \quad @ \text{ equilibrium}$$

$$\ln \frac{[\text{Na}^+]_{\text{in}}}{[\text{Na}^+]_{\text{out}}} = \frac{-zF(\phi_2 - \phi_1)}{RT}$$

Same as (a) $[\text{Na}^+]_{\text{in}} = [\text{Na}^+]_{\text{out}} e^{\frac{-zF(\phi_2 - \phi_1)}{RT}}$

- c. (10 pts) Suppose the membrane surface has a negative surface potential of $\Phi_0 = -75 \text{ mV}$ (e.g. Φ at membrane surface = $\Phi_1 - 75 \text{ mV}$ for the outer surface and for the inner surface ...). Under what circumstances would this surface potential effect $[\text{Na}^+]$ inside the vesicle? Hint: Equations given in problem 3 are useful and use equations to justify your answer.

Φ_0 will only affect $[\text{Na}^+]$ within the Debye length, L_D , which depends on the ionic strength of the solution:

$$L_D = \frac{10}{\sqrt{I}}$$

If the radius of the vesicle is larger than the Debye length, the $[\text{Na}^+]$ will only be affected within the Debye length. If the radius is similar to or smaller than the L_D , $[\text{Na}^+]$ will increase in the vesicle.