

Mechanics of Materials (CE130-I) Spring 2002

The First Mid-term Examination

Problem 1.

Derive the equilibrium equation for a two-dimensional infinitesimal element in the horizontal (x) direction. Note that the thickness of the element (z -direction) is taken as 1 (unit length), and X, Y are the body forces with the unit (force per unit volume). (20 points)

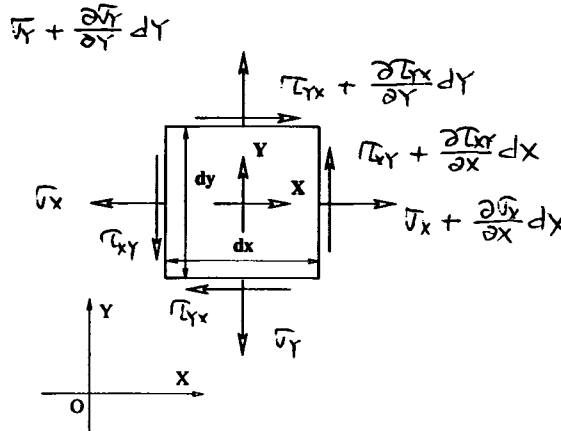


Figure 1: A 2D infinitesimal element

Problem 2

Suppose that an infinitesimal element ABCD deforms into ABC'D' as shown in the Figure 2. At point A, $u = v = 0$. Assume $\Delta > 0$. Find the shear strain at point A. (20 points)

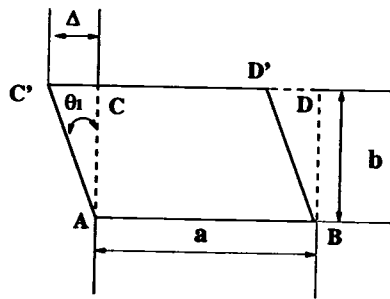


Figure 2: Shear deformation of an infinitesimal element

Problem 3

A wall bracket is constructed as shown in the Figure 3. All joints may be considered pin connected. There is an external force $P = 3\text{ kN}$ acting at point B. Find the reactions at point A and point C. (20 points)

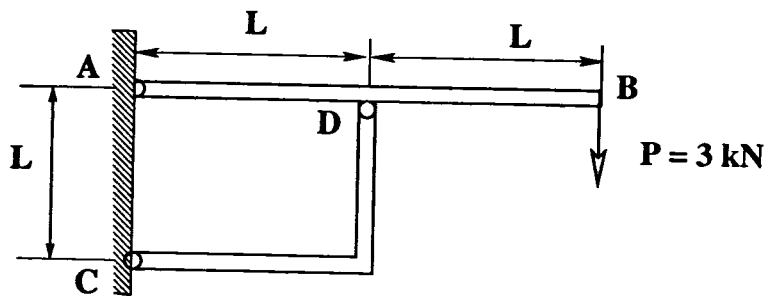


Figure 3: A two-bar bracket system

Problem 4

Consider a three elastic bar system (statically indeterminate) with external forces. Both the length and the flexibility of each bar are shown in Figure 4. There is a temperature rise, say Δt ($^{\circ}\text{C}$), and the coefficient of thermal expansion is α , and the thermal strain can be calculated by the formula $\epsilon_T = \alpha \Delta t$. Find the reaction forces R_1 , R_2 .

(20 points)

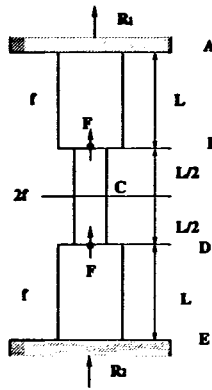


Figure 4: A three-bar statically indeterminate system

Problem 5

Two cylinder shafts made of the same material (G shear modulus), one solid cylinder with radius $c = R$ and one hollow cylinder with outer radius $c_o = R$ and inner radius $c_i = 0.5R$. They are subjected by an external torque, T_0 , as indicated in Figure 5.

(1) Find the maximum shear stress in each shaft and compare them.

(2) Find the angle-of-twist at the open end (where the external torque is applied) ϕ_b for each shaft and compare them. Hint:

$$\text{For a solid cylinder : } I_p = \frac{\pi d^4}{32} \quad (1)$$

$$\text{for a hollow cylinder : } I_p = \frac{\pi}{32}(d_o^4 - d_i^4), \quad d_o = 2c_o, \quad d_i = 2c_i \quad (2)$$

$$\tau_{\max} = \frac{Tc}{I_p}, \quad \Delta\phi = \frac{TL}{GI_p} \quad (3)$$

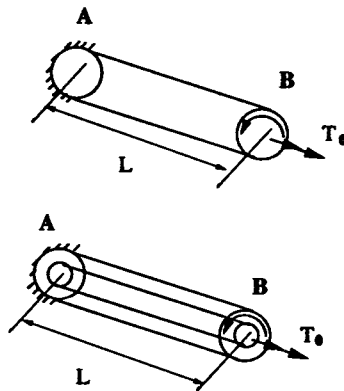


Figure 5: Torsion of two shaft systems