UNIVERSITY OF CALIFORNIA, BERKELEY College of Engineering

Mechanics of Materials (CE130-I) Fall 2005

The First Mid-term Examination

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Problem 1.

A circular stepped shaft has the dimensions shown in the figure. Use energy methof to determine the angle of twist at the loaded end. Shear modulus G is given.

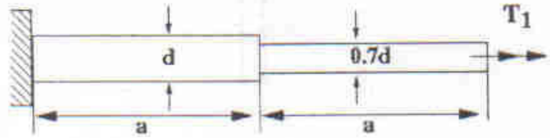

Figure 1: A 2D infinitesimal element

Hint:

$$\text{For a solid cylinder :} \quad I_p = \frac{\pi d^4}{32} \tag{1}$$

$$W_e = \frac{1}{2} M \Delta\phi, \quad U = \frac{T^2 L}{2GI_p} \tag{2}$$

Problem 2.

Derive the equilibrium equation for a two-dimensional infinitesimal element in the horizontal (x) direction. Note that the thickness of the element (z-direction) is taken as 1 (unit length), and X, Y are the body forces with the unit (force per unit volume). (20 points)

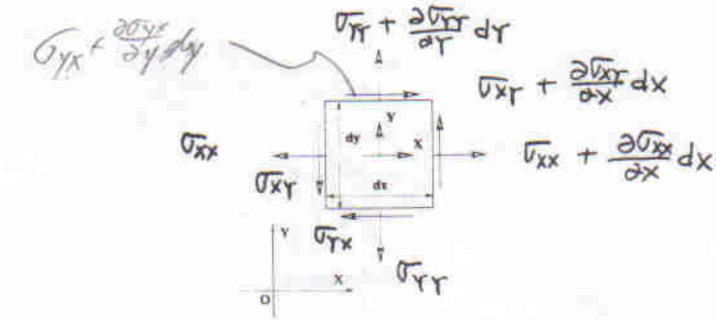

Figure 2: A 2D infinitesimal element

Problem 3

A wall bracket is constructed as shown in the Figure 3. All joints may be considered pin con-
ected. There is an external force $P = 3kN$ acting at point B. Find the reactions at point C. (20 oints)

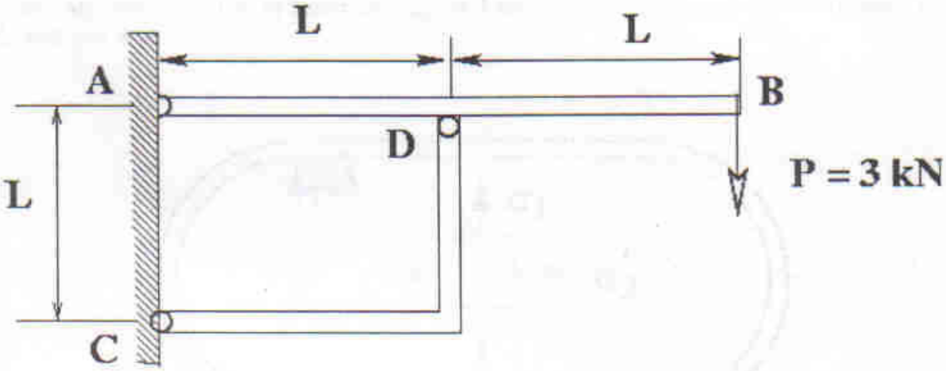

Figure 3: A two-bar bracket system

Problem 4

Consider a three elastic bar system (statically indeterminate) with external forces. Both the ength and the flexibity of each bar are shown in Figure 4. There is a temperature rise, say Δt (C^o), and the coefficient of thermal expansion is α, and the thermal strain can be calculated by the formula $\epsilon_T = \alpha\Delta t$. Find the reaction forces R_1, R_2. (20 points)

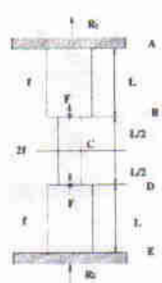

Figure 4: A three-bar statically indeterminate system

$$f = \frac{L}{EA}, \quad \text{and} \quad \Delta_T = \epsilon_T L \tag{3}$$

Problem 5

Consider a closed cylindrical steel presure vessel under internal pressure $p = 1.0MP_a$ as shown ı the figure.

(1) Find hoop stress σ_1 and longitudinal stress σ_2

(2) Find the hoop strain ϵ_1.

The average radius of the cylinder is, $r_{ave} = 1.0m$, and its thickness is $t = 10mm$. Let $E =$ $00GP_a$ and $\nu = 0.25$.

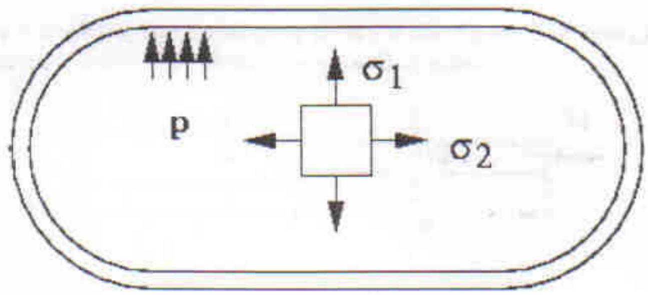

Figure 5: A cylindrical pressure vessel under internal pressure p.

Hint:

$$\sigma_1 = \frac{pr}{t} \quad (4)$$

$$\sigma_2 = \frac{pr}{2t} \quad (5)$$

ınd the generalized Hooke's law are

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \quad (6)$$

$$\epsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \quad (7)$$

$$\epsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E} \quad (8)$$

You may neglect the pressure in thickness direction.