CE152: Civil and Environmental Engineering Systems Analysis

Midterm

Prof. Madanat and Sengupta *Fall 04*

Problem 1 (6 points)

A standard problem in logistics is to determine the optimal size of shipments (i.e., the number of items) to ship from a warehouse to a retailer, assuming constant demand. The manager seeks the shipment size that provides the best tradeoff between the inventory (or holding) cost and the transport (or movement) cost.

The inventory cost (in $) is given by:
$C_i = AV$
where A is a variable cost per item in inventory, and V is the shipment size.

The transport cost (in $) is given by:
$C_t = B/V$
where B is a fixed cost.

Solve for the optimal shipment size.

Problem 2 (6 points)

As production manager of a factory your task is to determine the monthly production quantity for the next T months. The inventory I_t at the end of month t is:

$$I_t = I_{t-1} + P_{t-1} - d_{t-1}, \quad t = 1, \dots, T$$

where P_t is the monthly production quantity and d_t the monthly demand forecast. Your aim is to avoid fluctuation of the production quantity from month to month. This avoids labor shortages or under-utilization. Thus the production plan should minimize the cost function:

$$c_{inv} \sum_{t=1}^{T} I_t + c_{prod} \sum_{t=0}^{T-1} \left| P_{t+1} - P_t \right|$$

Formulate the production planning problem as a linear program. Include all reasonable constraints in your formulation. Do not include unnecessary constraints. Your factory is unable to hold more than K units of inventory at the end of a given month. Make sure you state the meaning of any additional variables you introduce.

Problem 3 (8 points)

Derive an optimal solution to the following linear program:

$$\min_{X_1, X_2, Y_1, Y_2} 2(X_1 + X_2)$$

subject to

$$\begin{aligned} Y_1 &= X_1 - 2 \\ X_1 + Y_2 &= X_2 + 3 \\ X_1 &\geq 0 \\ X_2 &\geq 0 \\ Y_1 &\geq 0 \\ Y_2 &\geq 0 \end{aligned}$$

Graphical or analytical derivations are acceptable.