UNIVERSITY OF CALIFORNIA, BERKELEY
FALL SEMESTER 2001

FINAL EXAMINATION

(CE130-2 Mechanics of Materials)

Problem 1: (15 points)

A pinned 2-bar structure is shown in Figure 1. There is an external force, $W = 5000N$, acting on the point C. (1) find internal axial forces for bar AC, BC; (2) find the normal stresses in bar AC and BC ($A_1 = A_2 = 0.01\ m^2$, and $E = 200$ MPa); (3) find the vertical displacement at nodal point C.

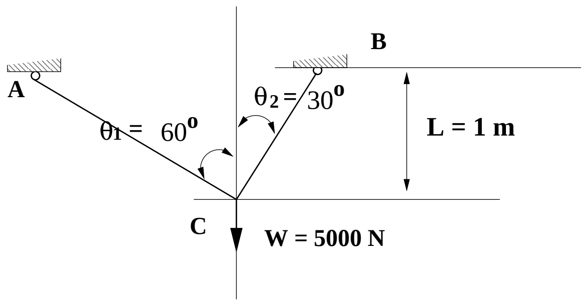

Figure 1: Schematic illustration of problem 1

(Hint: (1) Statics and equilibrium equations; (2) $\sigma = \dfrac{P}{A}$; (3) use Castigliano's second theorem, $\Delta_v = \dfrac{\partial U}{\partial W}$, or the energy method to find the vertical displacement at point C. The elastic potential energy in a single bar is: $U = \dfrac{P^2 L}{2EA}$, where P is the internal axial force, L is the length of the bar, E is the Young's modulus, and A is the cross section of the bar, and $W_e = \frac{1}{2} W \Delta_v$.)

Problem 2 (15 points)

A three-bar system as shown in Figure 2. The external force F is acting at the point C, i.e. the interface between the 2nd bar and the third bar. The system is statically indeterminate of rank one. (1) Find the reaction force at point A and B, i.e. R_A and R_B;

(2) Find the displacement at point C.

(Hint: use Castigliano's second theorem, or superposition method. The elastic potential energy in a single bar is: $U = \frac{P^2 L}{2EA}$, where P is the internal axial force, L is the length of the bar, E is the Young's modulus, and A is the cross section of the bar.)

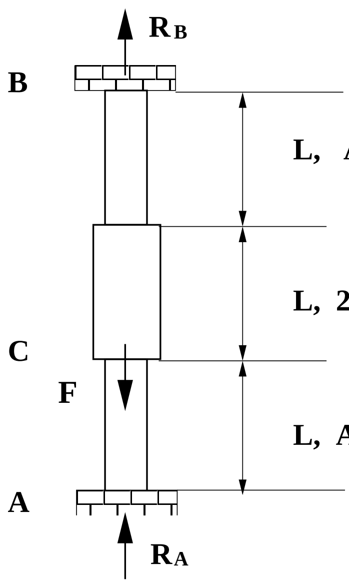

Figure 2: Schematic illustration of problem 2

Problem 3 (15 points)

The Cauchy stress tensor at a point is given in a matrix form (2D problem) as follows

$$\boldsymbol{\sigma} = \begin{pmatrix} 8 & 3 \\ 3 & 4 \end{pmatrix} \qquad \text{(MPa)}$$

a. find principal stresses σ_1, σ_2;

b. find the maximum shear stress;

c. draw Mohr's circle;

d. find the angle between the right face of the initial infinitesimal element and the plane on which principal stress σ_1 acting upon, and show the results on properly oriented element.

(Hint:

$$\begin{aligned}
\sigma_1 &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\
\sigma_2 &= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\
tan 2\theta_p &= \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}
\end{aligned}$$

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Problem 4 (10 points)

An infinitesimal triangular element is shown in the Figure 3. Let

$$\begin{aligned} \sigma_x &= \sigma \quad (1) \\ \sigma_y &= 3\sigma \quad (2) \\ \tau_{xy} &= \tau_{yx} = \tau \quad (3) \end{aligned}$$

and $\theta = 30^o$. Find the normal stress σ_θ and the shear stress τ_θ on the inclined plane in terms of σ and τ.

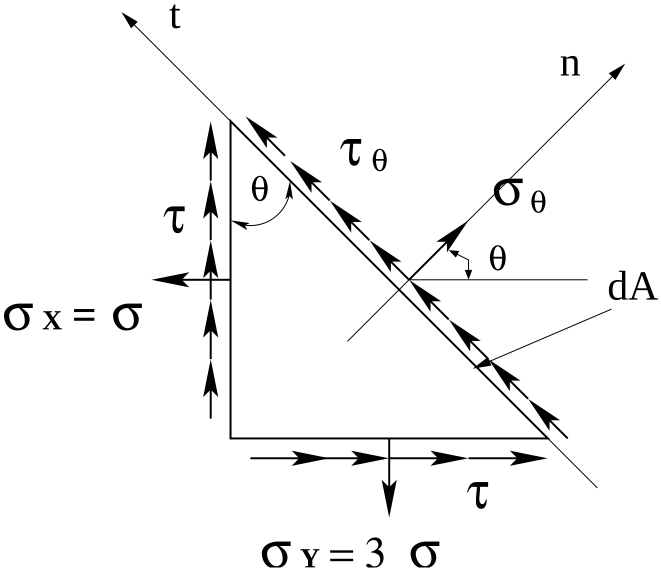

Figure 3: An infinitesimal triangle element

Problem 5 (10 points)

An over-hanging simply supported beam subjected a concentrated moment, $M_0 = 100.0$ N-m, and a distributed load $q_0 = -100\ \dfrac{N}{m}$, shown in Figure 4. Draw shear and moment diagrams.

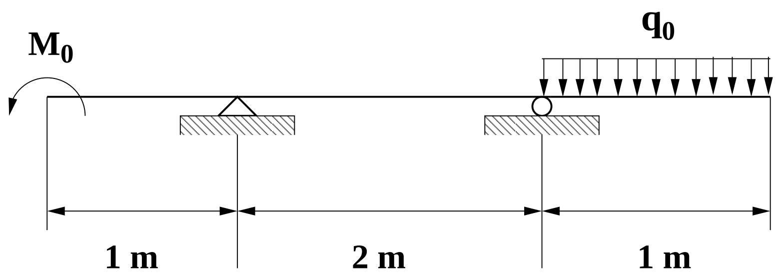

Figure 4: Simply supported beam with distributed load and concentrated moment.

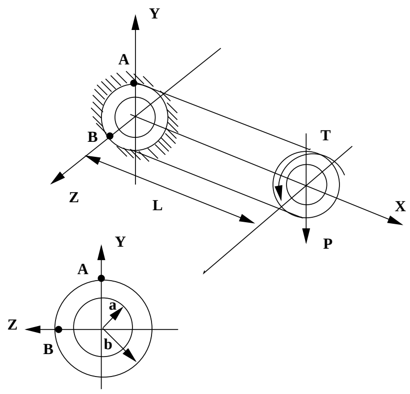

Figure 5: Illustration of Problem 6

Problem 6 (15 points)

A cantilever beam is made of a hollow cylinder with inner radius $a = 0.1m$ and outer radius $b = 0.2m$. The span of the beam $L = 2.0\ m$. There is a concentrated load, $P = 1000N$, and a concentrated torque, $T = 500$ N-m, acting on the free-end (as shown in Figure 5.). (1) Draw the moment diagram, shear diagram, and internal torque diagram; (2) Find the normal stress σ_x, shear stresses τ_{xy} and τ_{xz} at point A; (3) Find the normal stress σ_x and shear stress τ_{xy} at point B;

Hints: (1)

1. The flexural formula $\sigma_x = -\dfrac{M_z y}{I_z}$;
2. The torsion formula $\tau = \dfrac{T\rho}{I_p}$;
3. The shear formula $\tau = \dfrac{V(x)Q(y)}{I_z t}$

(2) For circular cross section

$$I_p = \frac{\pi R^4}{2}, \quad \text{where R is the radius of the circle} \tag{4}$$

$$I_z = \frac{1}{2} I_p \tag{5}$$

(3) For half annular cross section area (see Figure 5.)

$$Q(y) = \frac{2}{3}(b^3 - a^3) = \frac{2}{3}(b-a)(b^2 + ba + a^2) \tag{6}$$

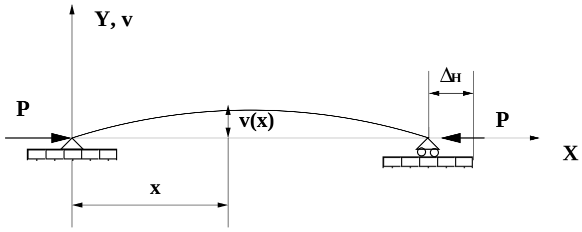

Figure 6: Problem 6

Problem 7 (10 points)

Using the neutral equilibrium condition, $\delta^2\Pi = 0$, determine the approximated critical buckling load P_{cr} for a simply supported (pinned) elastic column of constant EI. Assume that the deflected shape for a slightly bent column in a neighboring equilibrium position can be approximated as $v(x) = Ax(x - L)$, where A is the amplitude of the buckling mode, or a free variable controlling deflection. Compare your approximated result with the exact solution $P_{cr} = \dfrac{\pi^2 EI}{L^2}$.

(Hint:

(1) $\Pi = U - W_e$; (2) External work $W_e = P\Delta_v$, and

$$\Delta_v = \frac{1}{2}\int_0^L \left(\frac{dv}{dx}\right)^2 dx$$

The moment of the cantilever Euler column is $M(x) = -Pv(x)$ under the chosen coordinate system, and

$$U = \frac{1}{2EI}\int_0^L M^2(x)dx$$

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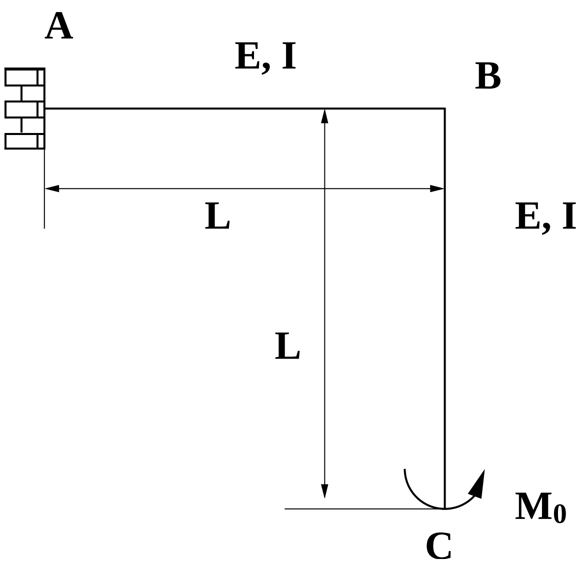

Figure 7: Problem 7

Problem 8 (10 points)

A planar frame ABC is subjected a concentrated moment, M_0, at point C as shown in Figure 7.

(a) Draw moment diagram;

(b) Find the cross section rotation at point C, i.e. θ_C ?

(Hint:

(1) use virtual force method,

$$\bar{1} \times \theta_C = \int \bar{m}_c(s) \frac{M(s)}{EI} ds$$

where s is a local coordinate; or

(2) use Castigliano's second theorem

$$\theta_C = \frac{\partial U}{\partial M_0}$$

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