① Flow in a square duct

a.) pressure drop without cylinder

$$A = (0.45m)^2 = 0.2025\,m^2$$

$$\overline{V} = Q/A = 3/0.2025 \,\tfrac{m}{s} = 14.815 \;m/s$$

$$R_H = \frac{A}{P} = \frac{b^2}{4b} = \frac{b}{4} = \frac{0.45m}{4} = 0.1125\,m$$

$$\therefore Re = (14.815)(4)(0.1125)/10^{-5} = 6.7\times 10^5$$

$$\frac{k_s}{4R_H} = \frac{0.8\times 10^{-3}}{4(0.1125)} = 0.0018 \longrightarrow f = 0.0235$$

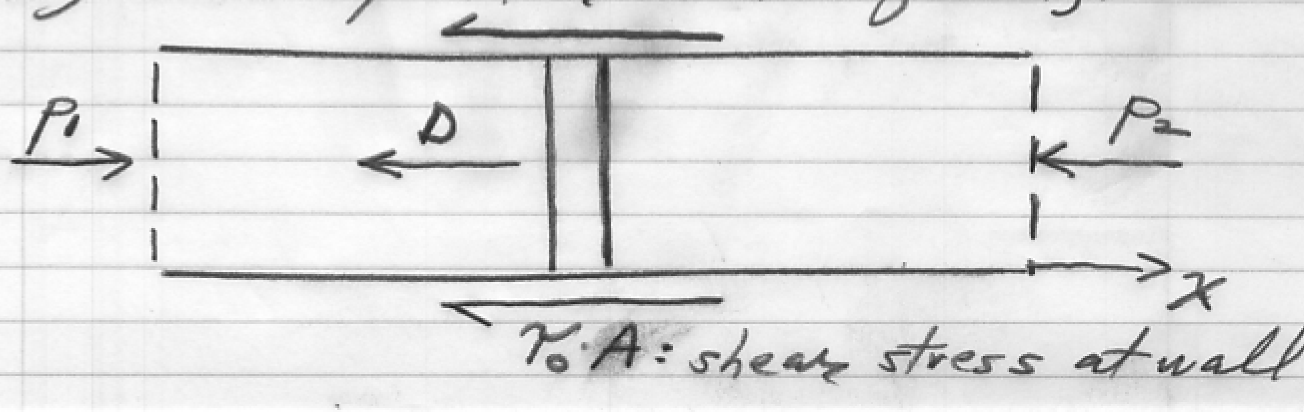
$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + \frac{fL}{4R_H}\frac{V_2^2}{2g}$$

$$V_1 = V_2,\; z_1 = z_2,$$

so $$\frac{\Delta p}{\underset{\underset{10\,N/m^3}{\parallel}}{\gamma}} = \frac{fL}{4R_H}\frac{V^2}{2g} = \frac{(0.0235)(3)(14.815)^2}{4(0.1125)\cdot 2(9.8)}$$

$$\Delta p = 17.54 \; Pa$$

b.) with cylinder: cons. of mom, FORCES



We take the control volume to be the air, and therefore the cylinder exerts drag in the direction opposite the flow.

$$\sum F_x = \dot{m}_{out} V_{out} - \dot{m}_{in} V_{in}$$

$$\dot{m}_{out} = \dot{m}_{in}, \quad V_{out} = V_{in}$$

so $\sum F = 0$

$$\sum F = (p_1 - p_2)\underbrace{A}_{b^2} - \tau_0 \underbrace{A_0}_{4bL} - D$$

τ_0:

Remember that $f \equiv \dfrac{\tau_0}{\frac{1}{8}\rho \bar{V}^2}$

so $\tau_0 = \dfrac{f}{8}\rho \bar{V}^2 = \dfrac{0.0235}{8}\left(\dfrac{10}{9.8}\right)(14.815)^2$

$= 0.658 \text{ Pa}$

$$A_0 = 4bL = 4(0.45)(3) = 5.4 \text{ m}^2$$

Drag: $C_D = \dfrac{D}{\frac{1}{2}\rho \bar{V}^2 A} \longrightarrow D = \frac{1}{2}\rho C_D \bar{V}^2 A$

$A = db = (0.03)(0.45) \text{ m}^2$

Cylinder $Re = \dfrac{\bar{V} d}{\nu}$

$= (14.815)(0.03)/10^{-5} = 4.4 \times 10^4$

From chart: $C_D = 1.1$

$\therefore\ D = \frac{1}{2} C_D \rho V^2 A$

$$= \frac{1}{2}(1.1)\left(\frac{10}{9.8}\right)(14.815)^2(0.03)(0.45)$$

$$= 1.66\ N$$

so $\Sigma F = \Delta p \cdot b^2 - \tau_0 A_0 - D = 0$

$$\Delta p = \frac{\tau_0 A_0 + D}{b^2}$$

$$= \frac{(0.658)(5.4) + 1.66}{(0.45)^2}$$

$$= 25.7\ Pa$$

c.) The assumption in part b.) was that the frictional head loss can still be estimated using the Darcy-Weisbach formula. In reality, the cylinder would affect the flow in the control volume, and the losses from friction would be different.

② FIRE BOAT PROBLEM

a.) First calculate flowrate.

We take the energy equation from the water free surface to the end of the hose, through the pump:

$$Z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_p = Z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + f\frac{L}{D}\frac{V^2}{2g}$$

$Z_1 = 0$, $Z_2 = 5m$

$p_1 = p_2 = 0$ (atmospheric)

$V_1 = 0$ (free surface not moving)

so $$h_p = Z_2 + \left(1 + f\frac{L}{D}\right)\frac{V^2}{2g}$$

or

$$V = \left[\frac{h_p - Z_2}{1 + f\frac{L}{D}}\right]^{1/2}(2g)^{1/2}$$

$$V = \frac{(30-5)^{1/2}(2\cdot 9.8)^{1/2}}{\left(1 + f\frac{4}{0.06}\right)^{1/2}} = \frac{22.1359}{(1+66.67f)^{1/2}}$$

$$\frac{k_s}{D} = \frac{0.6\times10^{-3}}{0.06} = 0.01$$

guess: $f\left(\frac{k_s}{D} = 0.01, Re = \text{High}\right) = 0.038$

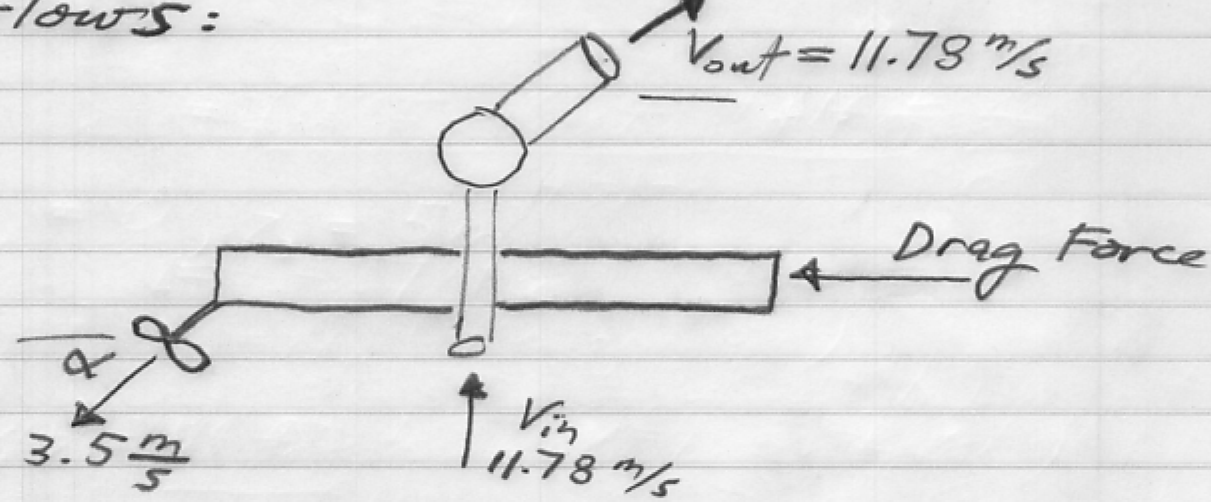
Then $V = \dfrac{22.1359}{(1 + 66.67(0.038))^{1/2}} = 11.78\,\frac{m}{s}$

CHeck $Re = 11.78(0.06)/10^{-6} = 7 \times 10^5$

$f = 0.038$ okay.

b.) Conservation of momentum

Flows:



FORCES: Drag from current $\leftarrow$ $(-x!)$

$$D = \tfrac{1}{2}\rho C_D V^2 A$$

$$= \tfrac{1}{2}(1000)(2.8)(0.15)^2(10)$$

$$= 315\,N$$

<u>X-Momentum</u>

$$\Sigma F_x = \Sigma \dot{m}_{out} V_{out_x} - \Sigma \dot{m}_{in} V_{in_x}$$

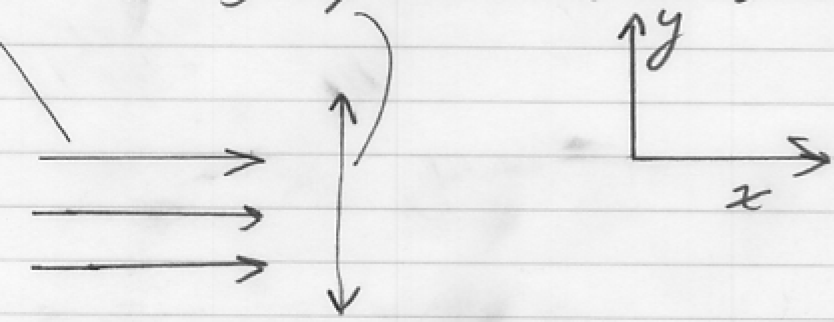
$$-D = \rho(11.78)\cdot\left(\frac{\pi(0.06)^2}{4}\right)(11.78)\cos 30^\circ$$

$$-315 \qquad - \rho(3.5)\left(\frac{\pi(0.3)^2}{4}\right)(3.5)\cos\alpha$$

$\cos\alpha = 0.76 \qquad \alpha = 41^\circ$

③ Oscillatory Flow

$$U_x = 1\ \tfrac{m}{s}\ ,\quad U_y = \pi \cos(2\pi t)\ m/s$$



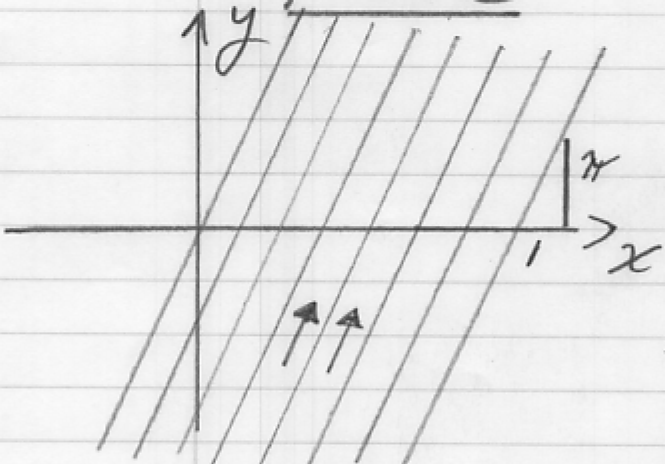
a.) $a_x = \dfrac{\partial U_x}{\partial t} = 0$

$$a_y = \frac{\partial U_y}{\partial t} = -2\pi^2 \sin(2\pi t)$$

b.) At $t = 2\,sec$: $U_x = 1\ m/s$

$U_y = \pi \cos(4\pi) = \pi\ \tfrac{m}{s}$

$t = 2\,sec$

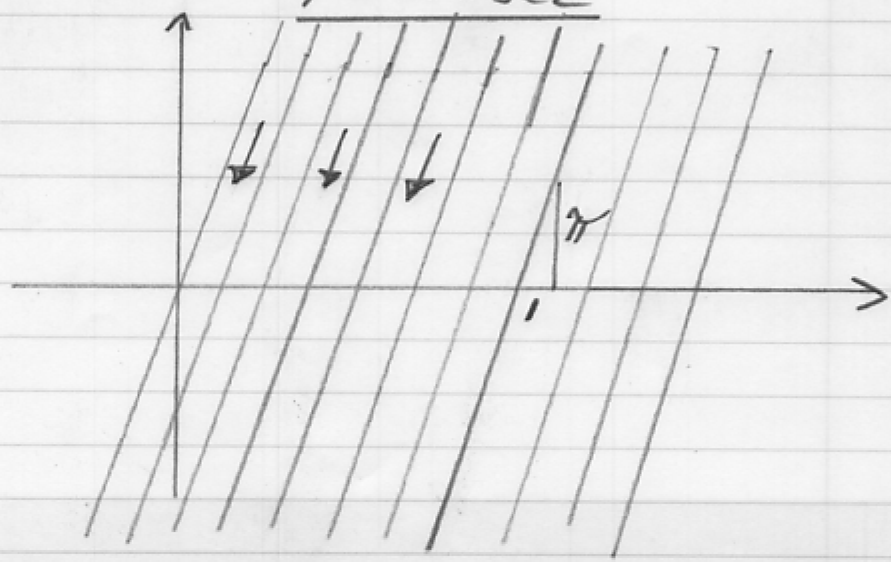


c.) $t = 2.5\,sec$

$U_x = 1\ m/s$

$U_y = \pi \cos(5\pi) = -\pi\ \tfrac{m}{s}$

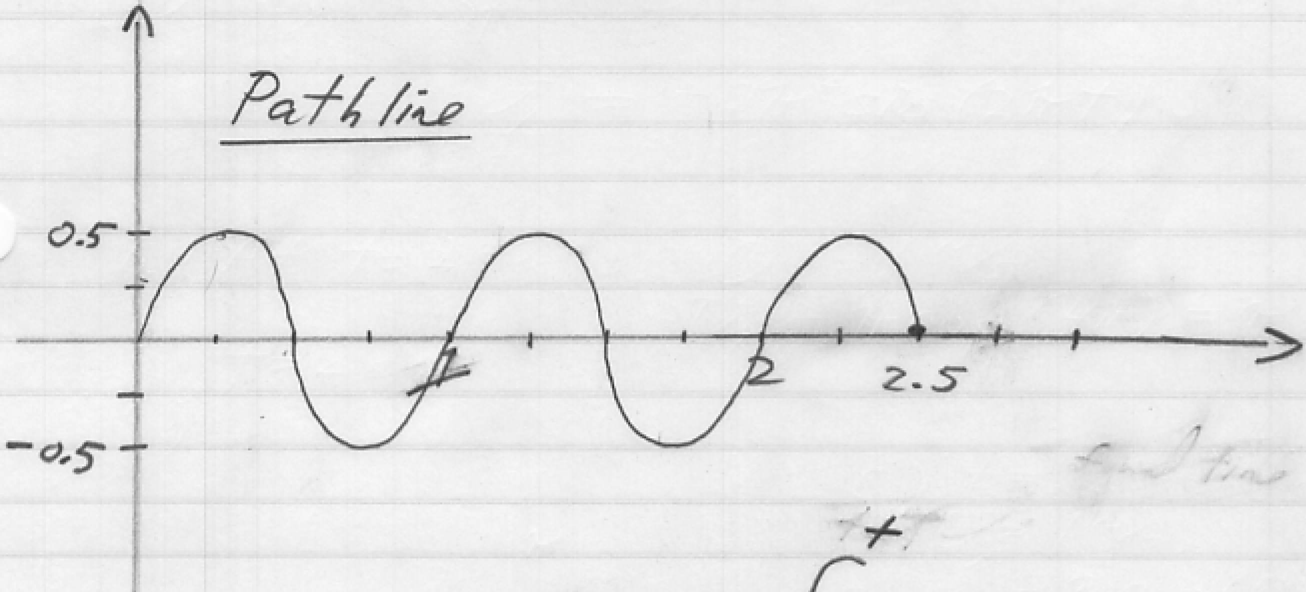
$t = 2.5\,sec$



d.) Pathline

You can think of the x-axis as time for the particle since $x = \frac{t}{v_x} = \frac{t}{1} = t$.

The cosine has period of 1 sec, which means that particle does one cycle of "back and forth" in 1 m:



since $v_y \equiv \frac{dy}{dt}$, $\int_{t=0}^{t} v_y(t)\,dt = y(t) - \underbrace{y(0)}_{0}$

$$y(t) = \int_0^t \pi \cos(2\pi t)\,dt = \frac{\pi}{2\pi} \sin(2\pi t)\Big|_0^t$$

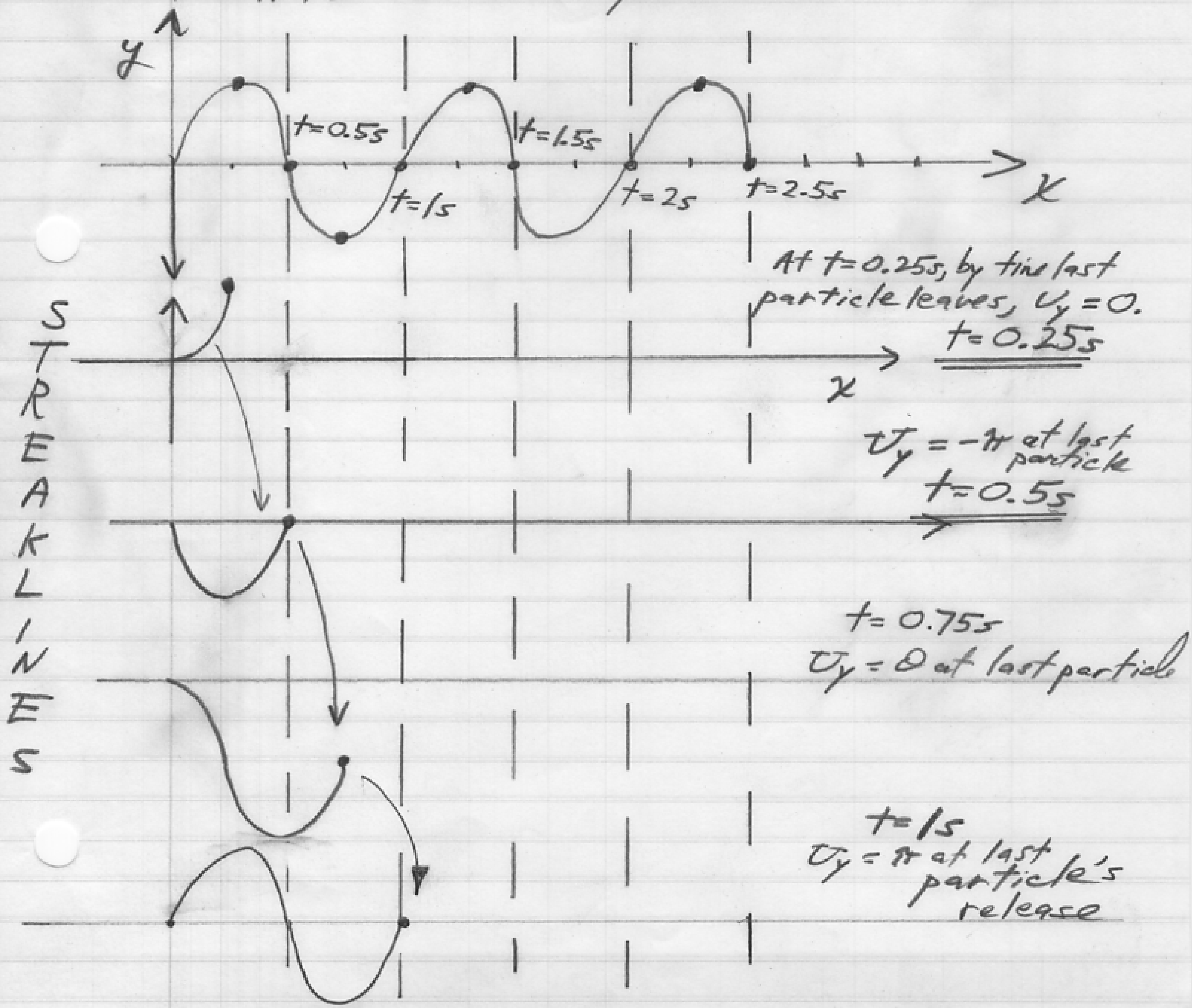
$$= \frac{1}{2} \sin(2\pi t)$$

$$v_x = \frac{dx}{dt} \longrightarrow x(t) = v_x t = 1 \cdot t$$

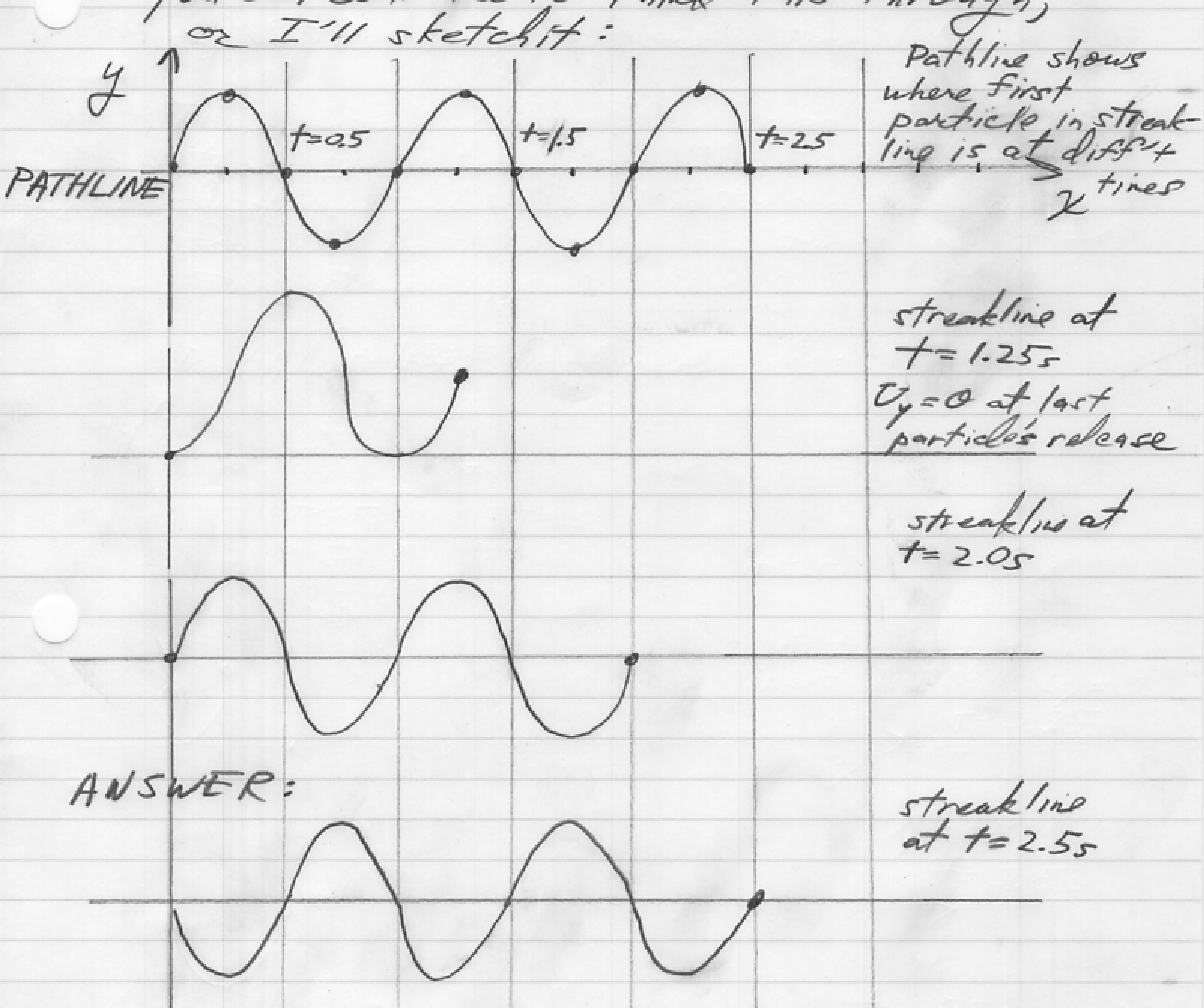
so $y(x) = \frac{1}{2} \sin(2\pi x)$ is our pathline e.) Streakline: this one is tough!

Think about the pathline, and the particles that come behind it. Remember that the first streakline particle, which is the particle used for the path line, is the first part of the streakline.

Pathline for first particle:



You can continue to think this through, or I'll sketch it:



You didn't have to draw all these out to figure it out, but I wanted to show you how it evolves.

release time

Mathematically: integrate from $t = t_r$ to t

A given particle's locations are:

$$y_p(t) = \frac{1}{2}\sin(2\pi t) - \frac{1}{2}\sin(2\pi t_r)$$

$$x_p(t) = (t - t_r)\cancel{v_x} = t - t_r$$

$$\text{so } t_r = t - x_p$$

streakline at some t $\longrightarrow y_p(t) = \frac{1}{2}\sin(2\pi t) - \frac{1}{2}\sin[2\pi(t - x_p)]$

We care about $t = 2.5s$:

$$y_p(t = 2.5s) = \cancel{\frac{1}{2}\sin(5\pi)}^{\,0} - \frac{1}{2}\sin[2\pi(2.5 - x_p)]$$

which is the plot that we obtained graphically.

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