University of California, Berkeley
College of Engineering

E117
Fall 2006

SECOND MIDTERM EXAM

1. (20 points) Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$$

Show that the eigenvectors are orthogonal.

2. (20 points) Check for path independence of the of the differential form

$$x\, y\, z^2\, dx + \tfrac{1}{2}\, x^2\, z^2\, dy + x^2\, y\, z\, dz\,.$$

If path independent, integrate from (0,0,0) to (a,b,c).

3. (20 points) Find the Laplace transform of the functions given below which are assumed to be zero outside the indicated intervals.

(a) $f(t) = t\,, \quad 0<t<1\,.$

(b) $f(t) = e^t\,, \quad 0<t<2\,.$

(Recall that $\mathcal{L}[f(t-a)u(t-a)] = e^{-as}\,\mathcal{L}[f]$).

4. (25 points) A harmonic oscillator is subjected to the effect of a forcing function including a delta function. Solve for motion y(t) of the oscillator subject to the given initial conditions.

$y'' + 5y = 25\,t - 100\,\delta(t-\pi), \qquad y(0) = -2, \qquad y'(0) = 5.$

(Recall $\mathcal{L}[f\,'] = s\mathcal{L}[f] - f(0)$, $\mathcal{L}[\delta(t-a)] = e^{-as}$).

5. (15 points) Let **A** be an n×n matrix.

(a) Express det(**A**) in terms of its eigenvalues.

(b) Using the result in (a) show that $\det(\mathbf{A}) \neq 0$ if and only if none of its eigenvalues is zero.

(c) If **A** is invertible, use the relation $\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$ to show that $\det(\mathbf{A}^{-1}) = 1/[\det(\mathbf{A})]$.