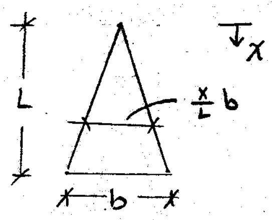
Midterm #1 | CE130 Chopra | Spring 2002

Prob 1



Axial force: $P_x = \gamma\left(\frac{1}{2}\cdot x\cdot\frac{x}{L}b\cdot 1\right) = \frac{1}{2}\frac{b\gamma}{L}x^2$

Cross-Section: $A_x = \left(\frac{x}{L}b\right)\cdot 1 = \frac{b}{L}x$

$$\Delta = \int_0^L \frac{P_x\,dx}{A_x E} = \int_0^L \frac{\frac{1}{2}\frac{b\gamma}{L}x^2\,dx}{\frac{b}{L}\cdot x E} = \frac{\gamma}{2E}\int_0^L x\,dx = \boxed{\frac{\gamma L^2}{4E}}$$

Prob 2

Stresses

$P_x = 40$ Kips $\quad A_x = 2\cdot 4 \quad \sigma_x = 5$ Ksi

$P_y = 48$ Kips $\quad A_y = 2\cdot 3 \quad \sigma_y = 8$ Ksi

$P_z = 36$ Kips $\quad A_z = 3\cdot 4 \quad \sigma_z = 3$ Ksi

E

$E = 2(1+\nu)G = 2(1+0.25)\,4\times10^6 = 10\times10^6$ psi $= 10\times10^3$ Ksi

ε_x

$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}[5 - .25(8+3)] = 2.25\times10^{-4}$

P_x'

$\varepsilon_x = \varepsilon_x' = \frac{\sigma_x'}{E} \qquad P_x' = \sigma_x' A_x$

$\sigma_x' = E\cdot\varepsilon_x = (10\times10^3\text{Ksi})(2.25\times10^{-4}) = 2.25$ Ksi

$P_x' = 2.25\text{ Ksi}\,(2\cdot 4) = \boxed{18\text{ Kips}}$

Prob 3

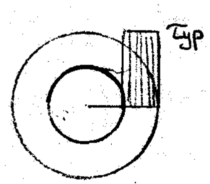
T_{yp}

$T_{yp} = \frac{\tau_{yp} I_p}{c}$

$I_p = \frac{\pi}{2}\left(c^4 - \left(\frac{c}{2}\right)^4\right) = \frac{15}{32}\pi c^4$

$T_{yp} = \frac{15}{32}\tau_{yp} c^3 \pi$

T_{ult}



$T = \int \tau\, 2\pi\rho^2 d\rho$

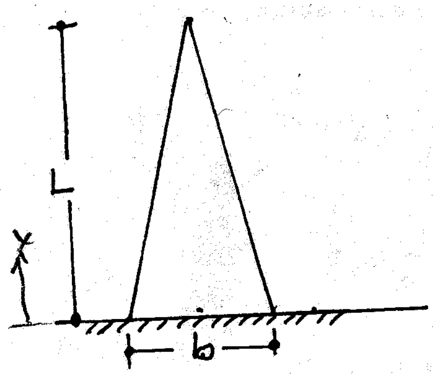
$T_{ult} = \int_{c/2}^{c} \tau_{yp}\, 2\pi\rho^2 d\rho = 2\pi\tau_{yp}\frac{1}{3}\rho^3\Big|_{c/2}^{c}$

$= 2\pi\tau_{yp}\frac{1}{3}\left(c^3 - \frac{c^3}{8}\right) = \frac{7}{12}\tau_{yp}\pi c^3$

$$\frac{T_{ult}}{T_{yp}} = \frac{\frac{7}{12}\tau_{yp}\pi c^3}{\frac{15}{32}\tau_{yp}\pi c^3} = \frac{7}{12}\cdot\frac{32}{15} = \frac{56}{45} = \boxed{1.24}$$

Tom Allen

1. The triangular bar shown is cut from a 1-in. thick plate. Determine the deflection of the top due to the weight of the bar. The unit weight of the material is γ; the elastic modulus is E. (Obvious assumption: Bar does not fall over)



$$\Delta = \int_0^L \frac{P}{AE}\,dx = \frac{1}{E}\int_0^L \frac{P}{A}\,dx$$

$$= \frac{1}{E}\int_0^L \frac{\frac{1}{2}\gamma t\left(\frac{L-x}{L}\right)b(L-x)}{t\left(\frac{L-x}{L}\right)b}\,dx$$

$$= \frac{1}{E}\int_0^L \frac{\gamma(L-x)}{2}\,dx$$

$$= \frac{\gamma}{2E}\int_0^L L-x\,dx$$

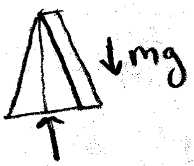
$$= \frac{\gamma}{2E}\left[Lx-\frac{x^2}{2}\right]_0^L$$

$$= \frac{\gamma}{2E}\left[L^2-\frac{L^2}{2}\right]$$

$$= \frac{\gamma}{2E}\left[\frac{L^2}{2}\right]$$

$$\Delta = \frac{\gamma L^2}{4E} \text{ downward}$$

Good!



$m = \gamma t \frac{1}{2} wh$

$w = \frac{(L-x)b}{L}$

$h = (L-x)$

$P = w = \gamma t \frac{1}{2}\left(\frac{L-x}{L}\right)b(L-x)$ ✓

$A = tw = t\left(\frac{L-x}{L}\right)b$ ✓

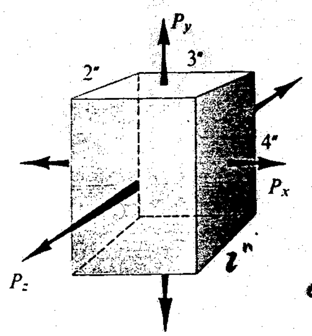
$P = w = \gamma V = \gamma t SA = \gamma t \frac{1}{2}\,\text{base}\cdot\text{height}$

$\text{base} = \left(\frac{L-x}{L}\right)b \quad \text{height} = L-x$

$P = \frac{1}{2}\gamma t\left(\frac{L-x}{L}\right)b(L-x)$

Tom Allen

2. A rectangular aluminum alloy block has the dimensions shown in the figure. The resultants of uniformly distributed stresses are $P_x = 40$ kips, $P_y = 48$ kips, and $P_z = 36$ kips. Determine the magnitude of a single system of tensile forces acting only in the x direction which would cause the same deformation in the x direction as the initial forces. Let $G = 4 \times 10^6$ psi, and $\nu = 0.25$.



$$\varepsilon_x = \frac{P_x}{A_x E} - \nu \frac{P_y}{A_y E} - \nu \frac{P_z}{A_z E}$$

$$E = 2G(1+\nu)$$
$$= 2(4\times10^6)(1+0.25)$$
$$E = 10\times10^6 \text{ psi}$$
$$= 10\times10^3 \text{ ksi}$$

$$\varepsilon_x = \frac{40}{2\cdot4\cdot10\times10^3} - 0.25 \frac{48}{2\cdot3\cdot10\times10^3} - \frac{36}{3\cdot4\cdot10\times10^3}$$

$$\varepsilon_x = 2.25\times10^{-4} \frac{in}{in}$$

$$\varepsilon_x = \frac{P_x}{A_x E} = 2.25\times10^{-4} \frac{in}{in}$$

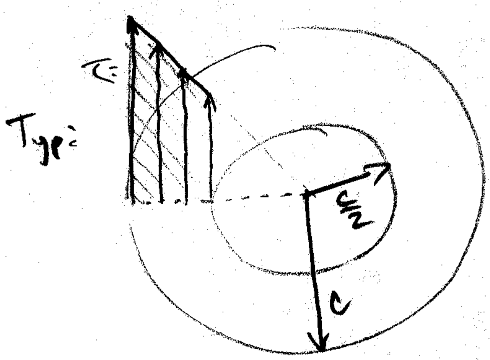
$$P_x = (2.25\times10^{-4})(A_x)(E)$$
$$P_x = (2.25\times10^{-4})(2\cdot4)(10\times10^3)$$

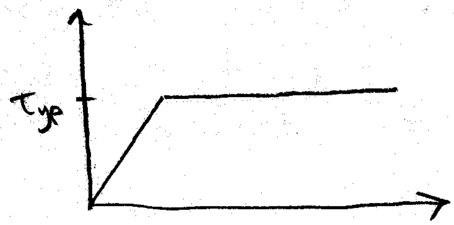
$$\boxed{P_x = 18 \text{ kips}}$$

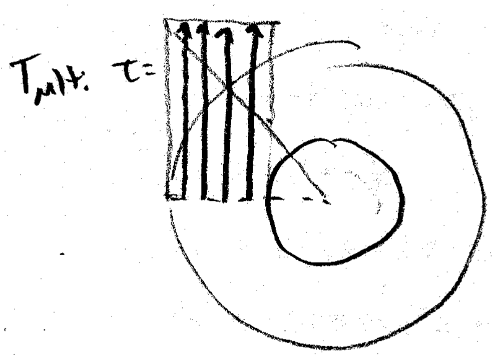
good

Tom Allen

3. Determine the ratio of the ultimate plastic torque T_{ult} to the yield torque T_{yp} for a tubular bar of circular cross-section. The inside radius is half of the outside radius c. Assume that the bar is made of an elastoplastic material.







$T = \int \tau \rho \, dA$

for yp $\tau = \tau_{yp} \frac{\rho}{c}$

$T = \int \tau_{yp} \cdot \frac{\rho}{c} \cdot \rho \, dA$

$= \int_{\frac{c}{2}}^{c} \tau_{yp} \cdot \frac{\rho}{c} \rho \cdot 2\pi\rho \, d\rho$

$= \int_{\frac{c}{2}}^{c} \tau_{yp} \frac{\rho^3}{c} \cdot 2\pi \, d\rho$

$= \frac{2\pi\tau_{yp}}{c} \int_{\frac{c}{2}}^{c} \rho^3 \, d\rho = \frac{2\pi\tau_{yp}}{c} \left. \frac{\rho^4}{4} \right|_{\frac{c}{2}}^{c}$

$= \frac{2\pi\tau_{yp}}{c} \left[\frac{c^4}{4} - \frac{(\frac{c}{2})^4}{4} \right]$

$= \frac{2\pi\tau_{yp}}{c} \left[\frac{16c^4}{64} - \frac{c^4}{64} \right] = \left(\frac{2\pi\tau_{yp}}{c} \right) \left(\frac{15}{64} c^4 \right)$

$T_{yp} = \frac{30\pi \tau_{yp} c^3}{64}$

for ultimate plastic torque

$\tau = \tau_{yp}$

$T_{ult} = \int \tau_{yp} \rho \, dA =$

$= \tau_{yp} \int \rho \cdot 2\pi\rho \, d\rho$

$= 2\pi \tau_{yp} \int_{\frac{c}{2}}^{c} \rho^2 \, d\rho$

$= 2\pi \tau_{yp} \left[\frac{\rho^3}{3} \right]_{\frac{c}{2}}^{c}$

$= 2\pi \tau_{yp} \left[\frac{c^3}{3} - \frac{(\frac{c}{2})^3}{3} \right]$

$= 2\pi \tau_{yp} \left[\frac{8c^3}{24} - \frac{c^3}{24} \right]$

$= 2\pi \tau_{yp} \left(\frac{7c^3}{24} \right)$

$T_{ult} = \frac{14\pi \tau_{yp} c^3}{24}$

$\frac{T_{ult}}{T_{yp}} = \frac{\frac{14\pi\tau_{yp}c^3}{24}}{\frac{30\pi\tau_{yp}c^3}{64}} = \frac{14 \cdot 64}{30 \cdot 24} = \frac{56}{45} = \boxed{1.244}$

great