University of California, Berkeley
College of Engineering

E117
Fall 2006

# FINAL EXAM

**1.** (20 points) Using separation of variables, solve the partial differential equation

$$u_t = \alpha\, u_{xx}$$

where $\alpha$ is a constant, subject to the boundary conditions $u(0,t) = u(\pi,t) = 0$ and the initial conditions $u(x,0) = sin(2x)$.

**2.** (20 points) The curved surface of a thin rod of length $L$ is insulated. Initially the temperature throughout the rod is $u(x, 0) = 100$. If the end of the rod at $x=0$ is insulated and the end at $x=L$ is kept at zero temperature for $t>0$, find the temperature in the rod for $t>0$.

**3.** (20 points) The torsional vibration of a uniform rod satisfies the differential equation

$$\theta_{tt} = a^2\, \theta_{xx}$$

where $\theta(x, t)$ is the angular displacement of the rod and $a^2$ is a constant. Consider a uniform rod of length $L$ which is fixed at one end and free at the other end. The appropriate boundary conditions are $\theta(0, t) = 0, \;\; \theta_x(L, t) = 0.$
If the initial angular displacement is $\theta(x, 0) = f(x)$ and the initial angular velocity is zero, find its subsequent motion.

**4.** (20 points) A square plate corresponds to the region $0 \le x \le a,\; 0 \le y \le a$. Its top and bottom surfaces are insulated. If the lower and upper sides at $y=0$ and $y=a$ are insulated, the side at $x=0$ is kept at zero temperature and the side at $x=a$ is kept at the temperature $u=f(y)$, find the steady-state temperature in the plate.

**5.** (20 points) The upper half of a sphere of radius $a$ is maintained at a constant temperature $u_0$ and the lower half at zero temperature. Find the steady-state temperature distribution $u(r, \varphi)$ inside the sphere.

**6.** (20 points) A rectangular membrane corresponds to the region $0 \le x \le a,\;\; 0 \le y \le b$. If it is given an initial displacement $u(x,y,0) = f(x,y)$ and released from rest, find its motion for $t>0$.