1a. 8
1b. 10
1c. 6
1d. 3
1e. 7
2ab. 17
2cd. 12
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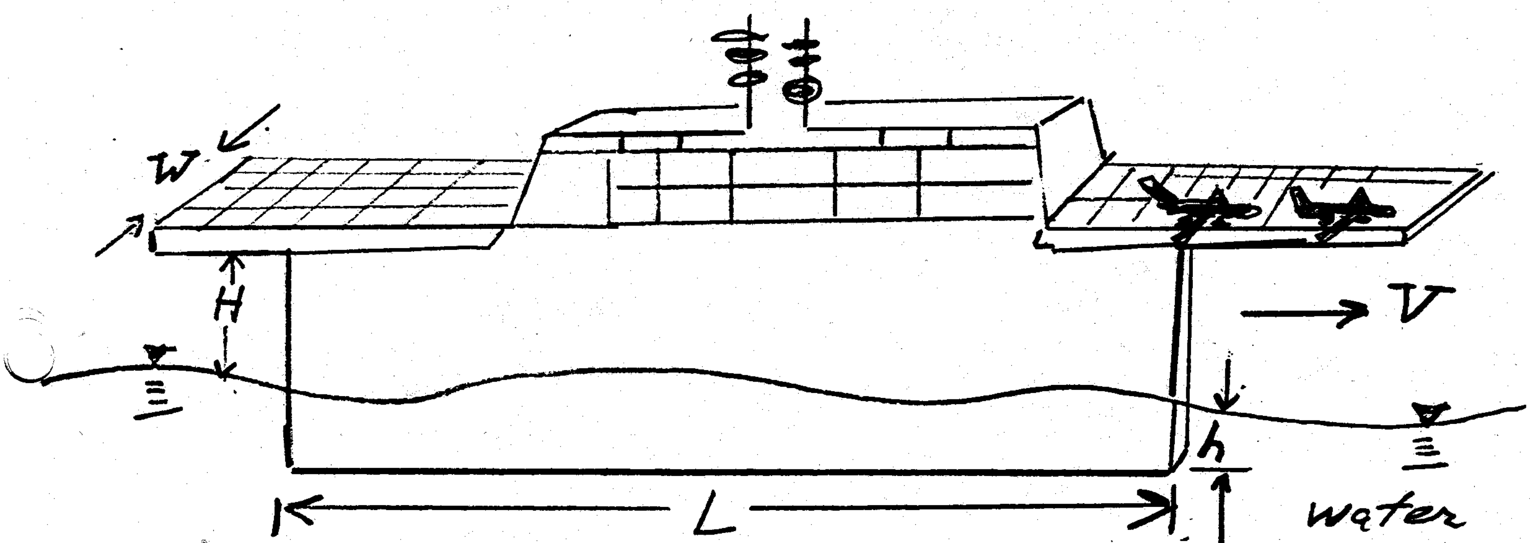
PROBLEM 1

A CV-61 aircraft carrier ship has the following dimensions:

- Maximum speed, V = 30 knots = 15.4m/s
- Length at waterline, L = 304m
- Width (at flight deck), W = 19.3m
- Height above waterline, H = 29.5m
- Draft below waterline, h=11m
- The viscosity of ocean water is $\nu=10^{-6}m^2/s$.

Great work, Irene!

The Ocean Engineering group at UC Berkeley maintains an experimental facility in Richmond. The facility has a towing tank where model ships can be towed at a maximum speed of 1.8m/s.



1a.) Experiments will be conducted in the towing tank to measure the drag on the ship, D, as a function of the ship geometry (see above for variables), ship speed V, fluid density ρ, fluid kinematic viscosity ν, and gravity g. Perform dimensional analysis on the problem and generate the nondimensional groups describing the flow. Your answer should be in the form $\pi_1 = f(\pi_2, \pi_3, \ldots)$, and you should rearrange the groups into the usual familiar groups. Neglect surface tension effects. (8pts)

$D = f(W, H, L, h, V, \rho, \nu, g)$

$\ell = L, \quad T = \frac{L}{V}, \quad M = \rho L^3$

$D \sim [\frac{M\ell}{T^2}] \quad \Pi_1 = D \frac{L^2}{V^2}\frac{1}{\rho L^3}\frac{1}{L} = \frac{D}{\rho V^2 L^2}$

$H \sim [\ell] \quad \Pi_2 = \frac{H}{L}$

$W \sim [\ell] \quad \Pi_3 = \frac{W}{L}$

$h \sim [\ell] \quad \Pi_4 = \frac{h}{L}$

$\nu \sim [\frac{\ell^2}{T}] \quad \Pi_5 = \nu \frac{L}{V}\frac{1}{L^2} = \frac{\nu}{VL} = \frac{1}{Re_L}$

$g \sim [\frac{\ell}{T^2}] \quad \Pi_6 = g\frac{L^2}{V^2}\frac{1}{L} = \frac{gL}{V^2} = \frac{1}{Fr^2}$

$$\boxed{\frac{D}{\rho V^2 L^2} = f\left(\frac{H}{L}, \frac{W}{L}, \frac{h}{L}, \frac{\nu}{VL}, \frac{gL}{V^2}\right)}$$

great!

1b.) Assume that the testing facility will be run at the maximum possible towing speed. *Ideally*, in order to achieve *full* dynamic similarity between the model and the prototype, what would be the values of
a.) the length of the model ship (5pts)
b.) the preferred viscosity of the fluid used in the model study (5pts)

p ~ prototype
m ~ model

$V_p = 15.4\ m/s$ $\quad V_m = 1.8\ \frac{m}{s}$

$L_p = 304\ m$ $\quad L_p = ?$

$\nu_p = 10^{-6}\ \frac{m^2}{s}$ $\quad \nu_m = ?$

GREAT! 10/10

Froude number similitude:

$$\frac{V_p}{\sqrt{gL_p}} = \frac{V_m}{\sqrt{gL_m}}$$

$$\frac{V_p^2}{L_p} = \frac{V_m^2}{L_m}$$

$$L_m = \frac{V_m^2}{V_p^2} L_p = \left(\frac{1.8}{15.4}\right)^2 (304\,m)$$

$$\boxed{L_m = 4.15m}$$ ✓

Reynold's number similitude:

$$\frac{V_p L_p}{\nu_p} = \frac{V_m L_m}{\nu_m}$$

$$\nu_m = \frac{V_m}{V_p}\frac{L_m}{L_p}\nu_p$$

$$\nu_m = \frac{1.8}{15.4}\ \frac{4.15}{304}\left(10^{-6}\ \frac{m^2}{s}\right)$$

$$\boxed{\nu_m = 1.60 \times 10^{-9}\ \frac{m^2}{s}}$$ ✓

1c.) If fresh water (ρ=1000kg/m^3) is used for the experiments in the hope that approximate similitude can be achieved, and a drag force D = 10 ~~14,000N~~ is measured in the experiment, what will be the drag be on the real ship? Assume that the density of ocean water is ρ=1025kg/m^3. (6pts)

C_p similitude

$$\frac{\Delta P_p}{\frac{1}{2}\rho_p V_p^2} = \frac{\Delta P_m}{\frac{1}{2}\rho_m V_m^2}$$

$$\frac{D_p}{A_p \rho_p V_p^2} = \frac{D_m}{A_m \rho_m V_m^2} \leftarrow \left(P = \frac{F}{A}\right)$$

$$D_p = \frac{A_p}{A_m}\frac{\rho_p}{\rho_m}\frac{V_p^2}{V_m^2} D_m$$

$$D_p = \left(\frac{304}{4.15}\right)^2 \frac{1025}{1000}\left(\frac{15.4}{1.8}\right)^2 (10N)$$ ✓

$$\boxed{D_p = 4.026 \times 10^6\ N = 4.026\ MN}$$ ✓

assuming geometric similitude

$$\frac{L_p}{L_m} = \frac{304}{4.15}$$

$$\frac{A_p}{A_m} = \frac{304^2}{4.15^2}$$

Great! 6/6

Next time, use:

$$\left(\frac{D}{\rho V^2 L^2}\right)_m = \left(\frac{D}{\rho V^2 L^2}\right)_p$$

1d.) How much of the measured drag on the **real** ship is due to skin friction? Assume that the flow around the ship's hull can be modeled as two growing boundary layers on each side of the ship that transition from laminar to turbulent (*neglect the friction on the underside of the ship*). The viscosity of ocean water is $\nu=10^{-6} m^2/s$. (8pts)

① $Re_L = \frac{VL}{\nu} = 1\times10^9 \rightarrow$ Turb!

② $C_f^r = \frac{0.523}{\ln^2(0.06 Re_L)} - \frac{\cancel{\quad}\, 1520}{Re_L}$

③ $F_s = \frac{1}{2}\rho h L V^2 C_f^r$

④ $F_s \times 2$

drag on one side of the ship:

$F_s = 0.664\, B \rho \nu V Re_L^{1/2}$

$= 0.664\, h \rho \nu V \left(\frac{VL}{\nu}\right)^{1/2}$

$= 0.664\, h \rho \nu^{1/2} V^{3/2} L^{1/2}$

$= 0.664\,(11m)(1025\,\frac{kg}{m^3})(10^{-6}\,\frac{m^2}{s})^{1/2}(15.4\,\frac{m}{s})^{3/2}(304m)^{1/2}$

$= 7889\,N$

total drag on ship $= 2(7889N) = \boxed{15777N}$

3/8

this is for laminar flow, and it is for C_f, not C_f^r. C_f is the local shear stress coefficient, which is used to calculate shear stress at a point. C_f^r is the average coefficient over the entire plate — used to calculate total force.

1e.) At sea, the ship drag is affected by ocean waves. For deep, small amplitude water waves, the additional parameter describing water waves is the wave period, T_0. The towing tank has a wavemaker to generate water waves. If a representative ocean wave period is T_0=*20s*, then what should the period of the waves be in the experiment? (7pts)

$\pi_7 = \frac{V_0 T_0}{L}$? $\left(\frac{VT_0}{L}\right)_m = \left(\frac{VT_0}{L}\right)_p$

$T = \frac{L}{V}$

$\frac{T_m}{T_p} = \frac{\frac{L_m}{V_m}}{\frac{L_p}{V_p}}$

7/7

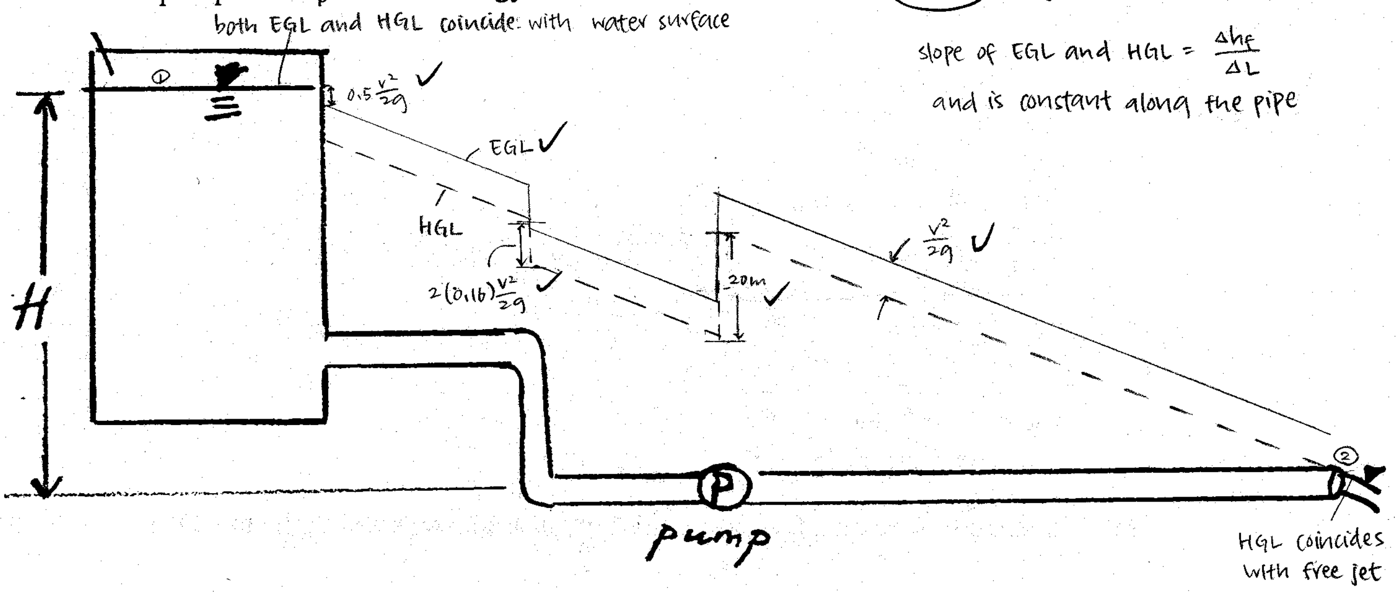
$T_m = T_p \frac{L_m}{L_p}\frac{V_p}{V_m} = (20s)\left(\frac{4.15}{304}\right)\left(\frac{15.4}{1.18}\right) = \boxed{2.34\,s}$ ✓

PROBLEM 2. Water ($\nu=10^{-6} m^2/s$ $\rho=1000 kg/m^3$) is fed from a reservoir into the pipe system as shown below:

$k_s = 0.26mm$

- Pipe is cast iron, with constant diameter D = 0.20*m*.
- Pipe entrance is sharp-cornered $k_E = 0.5$
- Pipe bends are smooth, with r/D=4: K=0.16
- Total length of pipe is L=20m.
- The pump adds h_p=20m of energy to the flow.

outstanding sketch! +1



2a.) Sketch the energy grade lines and hydraulic grade lines for the pipe system. Point out the important features, especially if you can't draw very well. (5pts) 6/5

2b.) What height H is required in the reservoir to maintain a flow of Q = 0.5m³/s? (11pts)

$V_{pipe} = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.5\frac{m^3}{s}}{\frac{\pi}{4}(0.20m)^2} = 15.9\frac{m}{s} = V_2$ (where water flows into the atmosphere)

$\frac{k_s}{D} = \frac{0.26}{200} = 0.0013$

$Re = \frac{VD}{\nu} = \frac{(15.9)(0.20)}{10^{-6}} = 3.2\times10^6$ → turbulent

From Moody diagram, f = 0.022

11/11

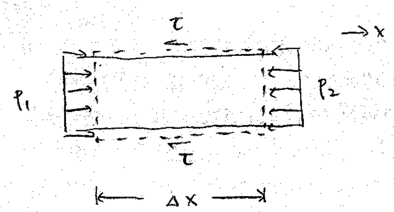
$$\frac{P_1}{\gamma} + z_1 + \frac{\alpha V_1^2}{2g} + h_p = \frac{P_2}{\gamma} + z_2 + \frac{\alpha V_2^2}{2g} + h_L$$

$$H + h_p = \frac{\alpha V_2^2}{2g} + \frac{V_2^2}{2g}\left(f\frac{L}{D} + k_E + k\right)$$

$$H = -20m + \frac{(15.9\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}\left[1 + 0.022\frac{20m}{0.2m} + 0.5 + 2(0.16)\right]$$

$\boxed{H = 31.8 m}$

2c.) In the fully-developed portion of the pipe, what is the shear stress at the wall of the pipe? (7pts)



assuming steady flow and constant cross-sectional area

$$\sum_{cs} \dot{m} V = 0$$

$$\sum F_x = P_1 A_1 - P_2 A_2 - \tau \pi D \Delta x = 0$$

$$\tau \cancel{\pi} \cancel{D} \Delta x = (P_1 - P_2) \frac{\cancel{\pi} D^{\cancel{2}}}{4}$$

7/7

using energy equation

$$\frac{P_1}{\gamma} + \cancel{z_1} + \frac{\alpha V_1^2}{2g} = \frac{P_2}{\gamma} + \cancel{z_2} + \frac{\alpha V_2^2}{2g} + h_L$$

$$(P_1 - P_2)\frac{1}{\gamma} = \cancel{\left(\frac{\alpha V_2^2}{2g} - \frac{\alpha V_1^2}{2g}\right)}^{0} + \frac{V_2^2}{2g} f \frac{\Delta x}{D}$$

$$(P_1 - P_2) = f \frac{\Delta x}{D} \frac{V_2^2}{2} \rho$$

$$\tau = (P_1 - P_2) \frac{D}{4 \Delta x}$$

$$\tau = f \frac{\cancel{\Delta x}}{\cancel{D}} \frac{V_2^2}{2} \rho \frac{\cancel{D}}{4 \cancel{\Delta x}}$$

$$\boxed{\tau = f \frac{V_2^2}{2} \frac{\rho}{4}}$$

$$\tau = 0.022 \left(\frac{(15.9 \frac{m}{s})^2}{2}\right) \frac{(1000 \frac{kg}{m^3})}{4}$$

$$\boxed{\tau = 695 \frac{N}{m^2}}$$ ✓

2d.) What is the boundary layer thickness in the fully-developed portion of the pipe? (5pts)

In the fully developed portion of the pipe, the boundary layer ceases to grow and maintains a constant thickness which is the radius of the pipe.

$$\delta = \frac{D}{2} = \boxed{0.10 m}$$

5/5 great!