Instructor: T.A. Morgan
University of California, Berkeley

Midterm #2

Name

Scoring

1		(20)
2		(20)
3		(10)
Total		(50)

Instructions

This exam is closed book. For reference, a formula sheet is attached. You must submit this formula sheet along with your exam. Please write your name at the top of all sheets submitted. Clearly state any assumptions you make beyond what is described in the problem. You may assume small angle approximations are valid. Partial credit may be awarded based on understanding of principles embedded in each problem. Therefore extraneous or irrelevant computations will *decrease* your overall score where the final answer is incorrect. If you need more space to write, use the back of the page.

The duration of the examination is 50 minutes.

Good Luck

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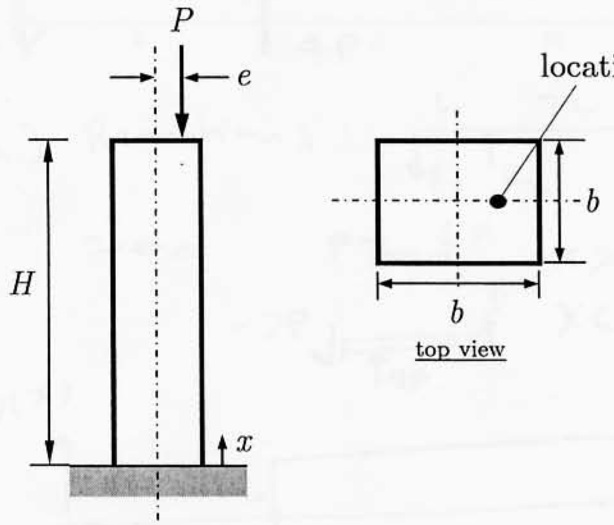
Problem 1 (20 points)

Consider the column shown below, having height H and *square* cross section (with each side having dimension b.) A load P is applied at the top, but is offset from the centroid of the section by an eccentricity e (as shown in the figure.) The material is brittle (like chalk) and is very weak in tension. Ignore the self-weight of the column. All answers should be expressed in terms of parameters given. Assume E.

a) Write the ordinary differential equations (ODE's) describing $u(x)$ and $v(x)$, where u is the axial displacement, and v is the transverse displacement. Include boundary conditions (BC's) for each. See the space provided below for your answer. Note: you are only asked to write the ODE's and BC's, **not** to solve them.

b) What is the maximum eccentricity e such that no tension stress occurs anywhere in the column?

$$\left[A = b^2 \quad I = \frac{1}{12} b^4 \right]$$



for 1-D

$$\frac{dP}{dx} + b_x = 0$$

$$P = \sigma A = EA\varepsilon = EAu'$$

$$\Rightarrow EAu'' + b_x = 0$$

For use in answering part a)

ODE for $u(x)$: $b^2 E u'' = 0$

BC's for $u(x)$: $u(0) = 0,\ b^2 E u'(H) = -P$

ODE for $v(x)$: $\frac{1}{12} b^4 E v'''' = 0$

BC's for $v(x)$: $v(0) = 0,\ v'(0) = 0,\ v'''(H) = 0,\ EIv''(H) = -Pe$

b) $\sigma_x = \frac{P}{A} - \frac{Mc}{I} = 0$ ← for no tension.

$$\frac{P}{b^2} = \frac{Pe(b/2)}{\frac{1}{12}b^4} \rightarrow \boxed{e = \frac{b}{6}}$$ units [L] ✓

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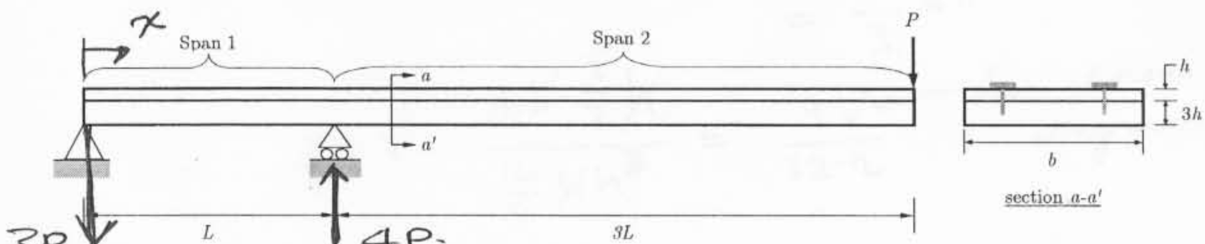
Problem 2 (20 points)

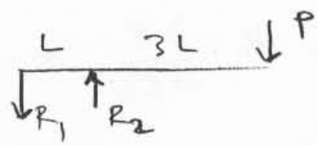
Consider the hanging cantilever beam with end load P, shown below. The beam is composed of two rectangular beams nailed together with two rows of nails. Each nail has an allowable shear force F_n. All answers should be expressed in terms of parameters given.

a) Draw and shear diagram $V(x)$ and the moment diagram $M(x)$ for this beam and the given loading.

b) For **each** of the two spans indicated, what is the required nail spacing? You should have two answers, one for Span 1, and another for Span 2.



a) Reactions:

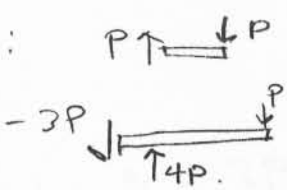


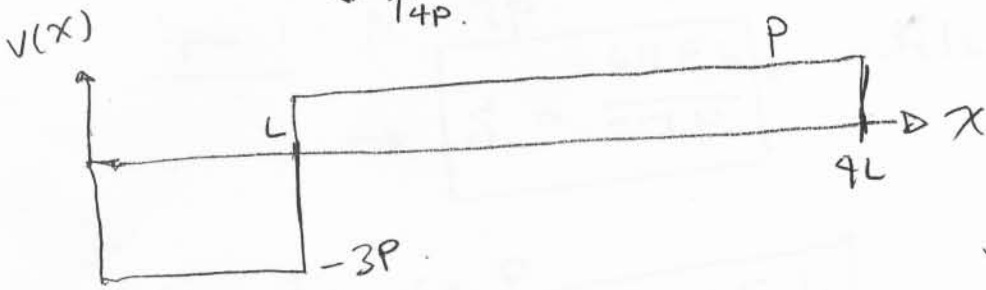
: $-P\cdot 4L + R_2 L = 0$

$R_2 = 4P.$

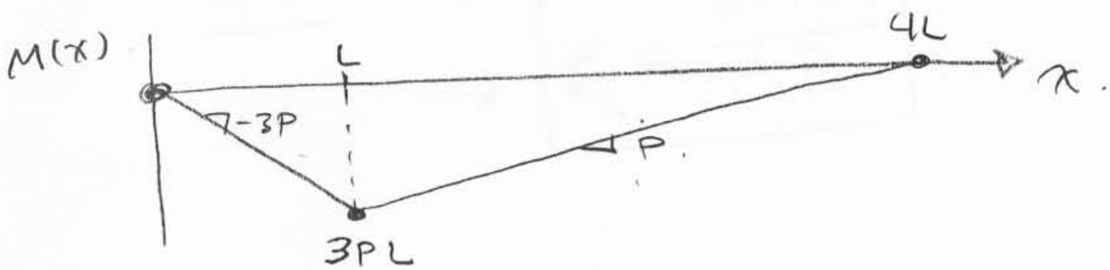
$R_1 = 3P.$

Shear:





$V = \frac{dM}{dx}.$



(see next page for (b))

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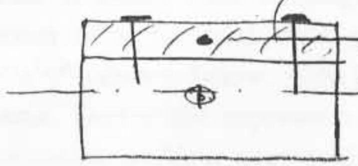
(Problem 2, continued)

b) 

$A^* = bh$

$\bar{y}^* = \frac{3}{2}h$ — N.A.

$q_{all} = \dfrac{2F_n}{s_R}$ ← two rows of nails. (s_R: spacing along x)

$$q = \frac{VQ}{I} = \frac{VA^*\bar{y}^*}{I}, \quad I = \frac{1}{12}b(4h)^3 = \frac{16}{3}bh^3$$

$$q = V \cdot \frac{bh \cdot \frac{3}{2}h}{\frac{16}{3}bh^3} = \frac{9V}{32 \cdot h}$$ ← from shear diagram.

$$q \le q_{all} \rightarrow \frac{9V}{32h} \le \frac{2F_n}{s}$$

$$s \le \frac{64F_n h}{9V} \quad (*)$$

Span 1: $V = 3P$

$$\rightarrow \boxed{s_1 = \frac{64F_n h}{27P}}$$ ← units [L] ✓

Span 2: $V = P$

$$\rightarrow \boxed{s_2 = \frac{64F_n h}{9P}}$$

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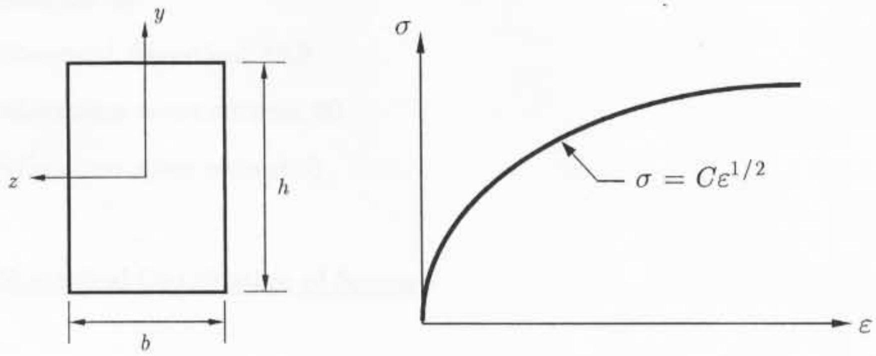
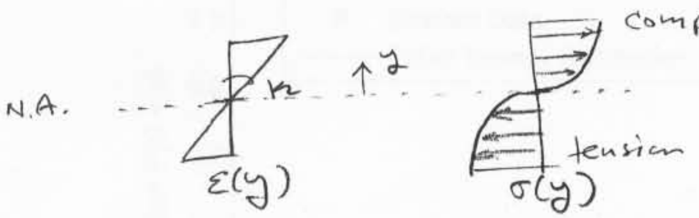
Problem 3 (10 points)

Consider a beam with rectangular cross section, shown below (left.) This beam is composed of a material with nonlinear stress-strain relation given by the equation $\sigma = C\varepsilon^{1/2}$ (shown below, right). The material response in compression and tension is the same. Derive the expression for the beam moment M about the z-axis in terms of the curvature κ. Your answer should be expressed in terms of parameters given.



Beam section — Material stress-strain relation



$$\sigma(y) = C\varepsilon^{1/2} = C(\kappa y)^{1/2} \qquad (\varepsilon = \kappa y)$$

$$M = -\int_A \sigma y \, dA = b\int_{-h/2}^{h/2} C(\kappa y)^{1/2} \cdot y \cdot dy$$

$$= 2bC\kappa^{1/2}\int_0^{h/2} y^{3/2}\, dy$$

$$= 2bC\kappa^{1/2}\, y^{5/2} \cdot \frac{2}{5}\Big|_0^{h/2}$$

$$= \frac{4}{5} bC\kappa^{1/2}\left(\frac{h}{2}\right)^{5/2} \qquad 2^{5/2} = 4\sqrt{2}$$

$$\boxed{M = \frac{Cbh^{5/2}}{5\sqrt{2}} \cdot \kappa^{1/2}}$$

units [F·L] ✓

Note: this is like EI, but for nonlinear M-κ relation.