

$$\frac{dy}{dx}$$

SL

Problem 1

$$\sum F_y = 0 \uparrow$$

$$\left(\bar{\sigma}_{yy} + \frac{\partial \bar{\sigma}_{yy}}{\partial y} dy\right) \cdot dx \cdot 1 - \bar{\sigma}_{yy} \cdot dx \cdot 1 + \left(\bar{\sigma}_{xy} + \frac{\partial \bar{\sigma}_{xy}}{\partial x} dx\right) \cdot dy \cdot 1 - \bar{\sigma}_{xy} \cdot dy \cdot 1 + \gamma dy dx \cdot 1 = 0$$

mult. ply everything  
by  $\frac{1}{dx dy}$

$$\frac{\left(\bar{\sigma}_{yy} + \frac{\partial \bar{\sigma}_{yy}}{\partial y} dy\right) - \bar{\sigma}_{yy}}{dy} + \frac{\left(\bar{\sigma}_{xy} + \frac{\partial \bar{\sigma}_{xy}}{\partial x} dx\right) - \bar{\sigma}_{xy}}{dx} + \gamma = 0$$

$$\boxed{\frac{\partial \bar{\sigma}_{yy}}{\partial y} + \frac{\partial \bar{\sigma}_{xy}}{\partial x} + \gamma = 0}$$

Problem 2.  $\delta_f = fP$   $\delta_T = \alpha(\Delta T)L$

①  $\Delta \leq (\alpha_1 L_1 + \alpha_2 L_2) \Delta T$

$\delta_{\text{thermal}} = (\alpha_1 L_1 + \alpha_2 L_2) \Delta T$

$(\delta_{\text{thermal}} - \Delta) + \delta_{\text{compression}} = 0$

$\delta_{\text{compression}} = -f_1 P_1 - f_2 P_2$

$P_2 = P_1 = R_c$

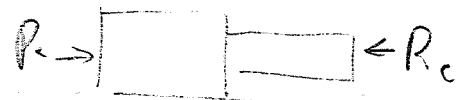
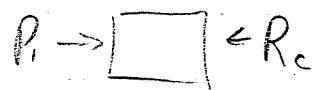
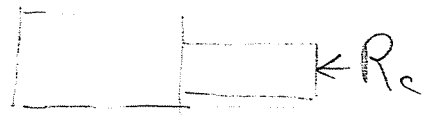
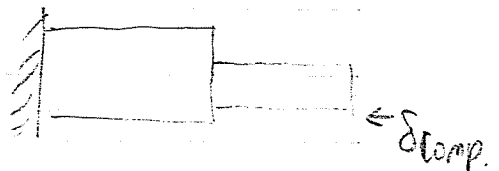
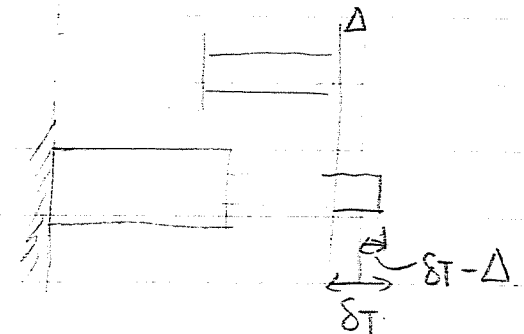
$\delta_{\text{compression}} = -R_c(f_1 + f_2)$   
↑ denote compression

$(\delta_{\text{thermal}} - \Delta) + \delta_{\text{compression}} = 0$

$(\alpha_1 L_1 + \alpha_2 L_2) \Delta T - \Delta - R_c(f_1 + f_2) = 0$

$R_c(f_1 + f_2) = (\alpha_1 L_1 + \alpha_2 L_2) \Delta T - \Delta$

$R_c = \frac{(\alpha_1 L_1 + \alpha_2 L_2) \Delta T - \Delta}{f_1 + f_2}$



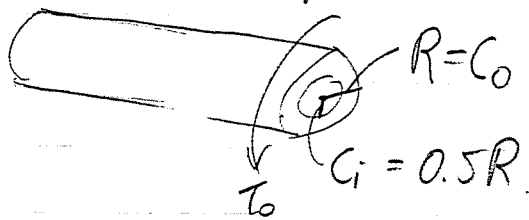
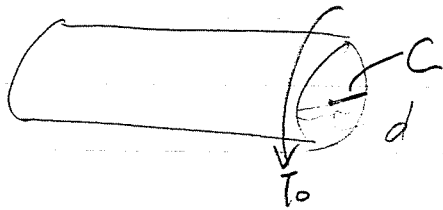
$R_A = R_c$   
 $R_A = \frac{(\alpha_1 L_1 + \alpha_2 L_2) \Delta T - \Delta}{f_1 + f_2}$

Problem 2

$$\textcircled{2} \Delta > (d_1 L_1 + d_2 L_2) \Delta T$$

Since the gap is big enough for thermal expansion, therefore, there will be no compression force or reaction needed

$$\therefore R_C = R_A = 0$$

Problem 3

①  $\tau_{\max \text{ solid}} = \tau_{\max \text{ hollow}}$

$$\tau_{\max \text{ solid}} = \frac{T_c}{J} = \frac{T_0 l}{\frac{\pi}{2} c^4} = \frac{2T_0}{\pi c^3}$$

$$\tau_{\max \text{ hollow}} = \frac{T_{c_0}}{\frac{\pi}{2} (c_0^4 - c_i^4)}$$

$$\frac{2T_0}{\pi c^3} = \frac{2T_0 R}{\pi (R^4 - (0.5R)^4)}$$

$$\tau_{\max H} = \frac{T_0 R}{\frac{\pi}{2} (R^4 - (0.5R)^4)}$$

$$\frac{1}{c^3} = \frac{R}{R^4 - 0.0625R^4} \Rightarrow R^4 - 0.0625R^4 = c^3 R$$

$$R^3 - 0.0625R^3 = c^3$$

$$c = \sqrt[3]{R^3 - 0.0625R^3}$$

$$c = \sqrt[3]{0.9375R^3} = R \sqrt[3]{0.9375}$$

$$c \approx 0.9787 R$$

Problem 3.

$$\textcircled{2} \quad \phi_S = \phi_H$$

$$\phi_S = \frac{T_0 \cdot L}{G_1 \cdot \frac{\pi}{2} (c^4)}$$

$$\phi_H = \frac{T_0 \cdot L}{G_2 \cdot \frac{\pi}{2} (C_0^4 - C_1^4)}$$

$$\phi_S = \frac{T_0 \cdot L}{G_1 \cdot \frac{\pi}{2} \left(\frac{d}{2}\right)^4} = \frac{T_0 \cdot L}{G_1 \cdot \frac{\pi}{32} d^4}$$

$$\phi_H = \frac{T_0 \cdot L}{G_2 \cdot \frac{\pi}{2} (R^4 - (0.5R)^4)}$$

$$\frac{T_0 L}{G_1 \cdot \frac{\pi}{32} d^4} = \frac{T_0 L}{G_2 \cdot \frac{\pi}{2} (R^4 - 0.5R)^4}$$

$$\frac{32}{G_1 d^4} = \frac{2}{G_2 (R^4 - 0.0625R^4)}$$

$$\frac{16}{32} G_2 (R^4 - 0.0625R^4) = G_1 d^4$$

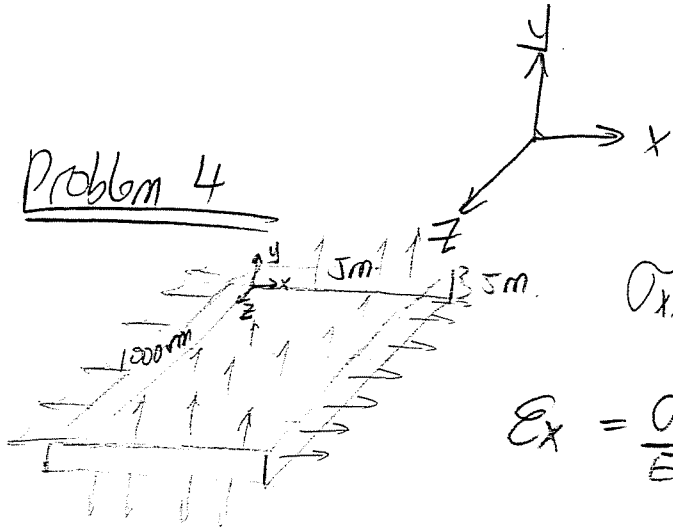
$$d^4 = \frac{16 G_2}{G_1} (R^4 - 0.0625R^4)$$

$$d = \sqrt[4]{\frac{16 G_2}{G_1} R^4 (0.9375)} \quad \left(0.9375 = \frac{15}{16}\right)$$

$$d = \sqrt[4]{\frac{G_2}{G_1} R^4 \frac{15}{16} \cdot 16}$$

$$\boxed{d = R \sqrt[4]{\frac{G_2 \cdot 15}{G_1}}} \quad \text{or} \quad \boxed{d \approx 1.977 R \sqrt[4]{\frac{G_2}{G_1}}}$$

Problem 4



$\sigma_{xx} = 5 \text{ MPa}$        $\sigma_{yy} = 10 \text{ MPa}$

$$\epsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E}$$

$$\epsilon_{zz} = -\nu \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}$$

$$0 = -\nu \sigma_{xx} - \nu \sigma_{yy} + \sigma_{zz}$$

$$\sigma_{zz} = \nu (\sigma_{xx} + \sigma_{yy})$$

$$\sigma_{zz} = 0.3 (5 \text{ MPa} + 10 \text{ MPa})$$

$$\sigma_{zz} = 4.5 \text{ MPa}$$

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu \sigma_{yy}}{E} - \frac{\nu \sigma_{zz}}{E}$$

$$\epsilon_{xx} = \frac{1}{E} (\sigma_{xx} - \nu \sigma_{yy} - \nu \sigma_{zz})$$

$$\epsilon_{xx} = \frac{1}{100 \text{ MPa}} (5 \text{ MPa} - 0.3 \cdot 10 \text{ MPa} - 0.3 \cdot 4.5 \text{ MPa})$$

$$\epsilon_{xx} = \frac{1}{100 \text{ MPa}} \cdot 0.65 \text{ MPa} \quad \epsilon_{xx} = 0.0065$$

$$\epsilon_{yy} = -\frac{\nu \sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \frac{\nu \sigma_{zz}}{E} = \frac{1}{100 \text{ MPa}} (10 \text{ MPa} - 0.3 \cdot 5 \text{ MPa} - 0.3 \cdot 4.5 \text{ MPa})$$

$$\epsilon_{yy} = \frac{1}{100 \text{ MPa}} \cdot (7.15 \text{ MPa}) = 0.0715$$