

Mechanics of Materials (CE130-I) Fall 2006

The First Mid-term Examination

Problem 1. (25 points)

Derive the equilibrium equation for a two-dimensional infinitesimal element in the vertical (Y) direction. Note that the thickness of the element (z-direction) is taken as 1 (unit length), and X, Y are the body forces with the unit of force per unit volume.

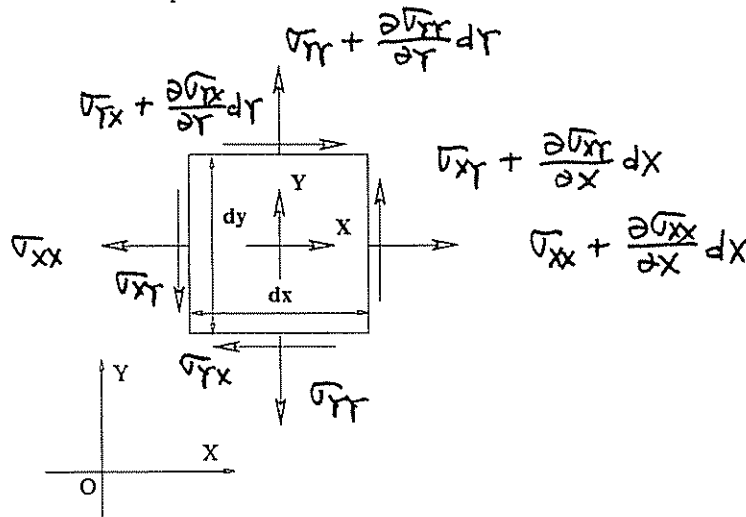


Figure 1: A 2D infinitesimal element

Problem 2 (25 points)

Consider the following two-bar system (Figure 2). The flexibilities of two elastic bars are given as f_1 and f_2 , the lengths of the two bars are L_1 and L_2 , and the thermal expansion coefficients of the two bars are given as α_1 and α_2 . The right end of the second bar has a distance Δ from the wall. There is a temperature increase of ΔT . Find the reaction forces at the point A, i.e. R_A , and at the point C, i.e. R_C , for two different cases:

- (1) $\Delta \leq (\alpha_1 L_1 + \alpha_2 L_2) \Delta T$;
- (2) $\Delta > (\alpha_1 L_1 + \alpha_2 L_2) \Delta T$.

Hints: $\Delta_P = fP$ and $\Delta_T = \alpha L \Delta T$.

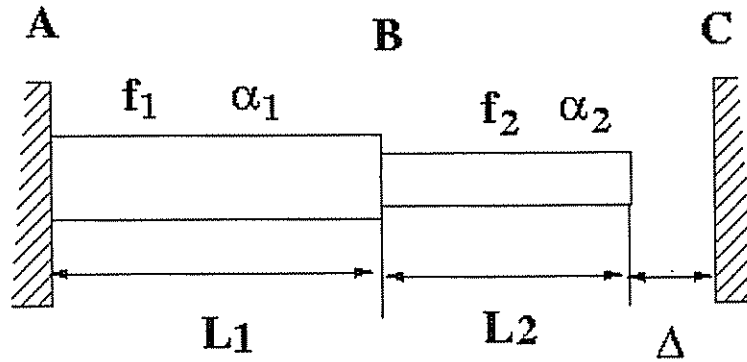


Figure 2: A possible statically Indeterminate System

Problem 3 (25 points)

Two cylindrical shafts are made of different materials, i.e. shear modulus $G_1 \neq G_2$, but have the same length L . One solid cylinder of diameter $2c = d$, and the other is a hollow cylinder with outer radius $c_o = R$ and inner radius $c_i = 0.5R$. They are subjected to the same external torque, T_0 , as indicated in Figure 3.

- (1) If the maximum shear stress in both shafts is the same, find the relationship between $c \sim R$?
- (2) If the angle-of-twist at the free end (where the external torque is applied) ϕ is the same for both shafts, find the relationship of $d \sim R$?

Hints:

$$\text{For a solid cylinder : } J = \frac{\pi d^4}{32} \quad (1)$$

$$\text{For a hollow cylinder : } J = \frac{\pi}{32}(d_o^4 - d_i^4), \quad d_o = 2c_o, \quad d_i = 2c_i \quad (2)$$

$$\tau_{\max} = \frac{Tc}{J}, \quad \phi = \frac{TL}{GJ} \quad (3)$$

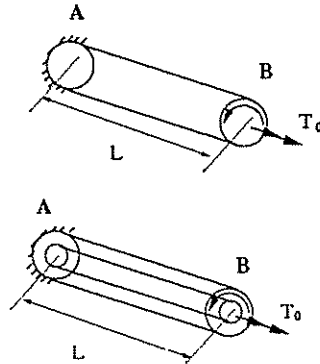


Figure 3: Torsion of two shafts

Problem 4 (25 points)

Consider a very long (1000 meters in z-direction) concrete block with its both ends fixed. The cross section of the concrete block (section in x-y plane) is a 5 meter square. Suppose that in x-y plane, the block is subjected biaxial tensile stress load, namely, $\sigma_{xx} = 5MP_a$ and $\sigma_{yy} = 10MP_a$. This is a typical *plane strain* state ($\epsilon_{zz} = 0$). Let $E = 100MP_a$ and Poisson's ratio $\nu = 0.3$. Find σ_{zz} , ϵ_{xx} , and ϵ_{yy} .

Hints: The equations of the generalized Hooke's law are

$$\begin{aligned}\epsilon_{xx} &= \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{yy}}{E} - \nu \frac{\sigma_{zz}}{E} \\ \epsilon_{yy} &= -\nu \frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \nu \frac{\sigma_{zz}}{E} \\ \epsilon_{zz} &= -\nu \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E}\end{aligned}$$