Midterm 2 Solutions (Section I)

Problem 1.

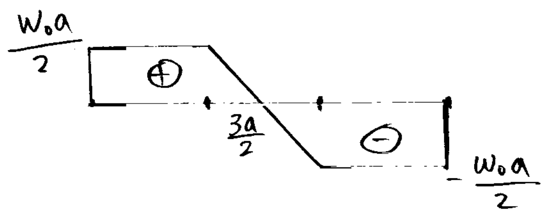
(a) $q(x) = EIv^{(4)} = -w_0 \langle x-a\rangle^0 + w_0 \langle x-2a\rangle^0$

$V(x) = EIv''' = -w_0 \langle x-a\rangle + w_0 \langle x-2a\rangle + C_1$

$M(x) = EIv'' = -\frac{w_0}{2}\langle x-a\rangle^2 + \frac{w_0}{2}\langle x-2a\rangle^2 + C_1 x + C_2$

$V(0) = \frac{w_0 a}{2} \Rightarrow C_1 = \frac{w_0 a}{2}$

$M(0) = 0 \Rightarrow C_2 = 0$



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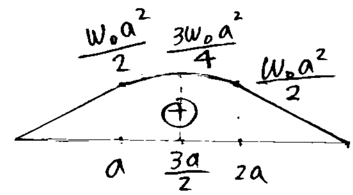


moment

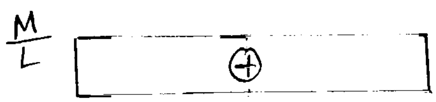
(b) $q(x) = EIv^{(4)} = -M\langle x-L/2\rangle^{-2}$

$V(x) = EIv''' = -M\langle x-L/2\rangle^{-1} + C_1$

$M(x) = EIv'' = -M\langle x-L/2\rangle^0 + C_1 x + C_2$

$V(0) = \frac{M}{L} \Rightarrow C_1 = \frac{M}{L}$

$M(0) = 0 \Rightarrow C_2 = 0$



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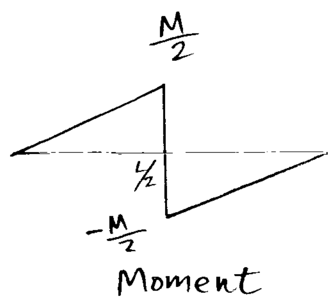


Moment

problem 2.

$Q = 500 \times 50 \times \left(\frac{500}{2} + \frac{50}{2}\right) \times 10^{-9} = 6.875 \times 10^{-3}\ m^3$

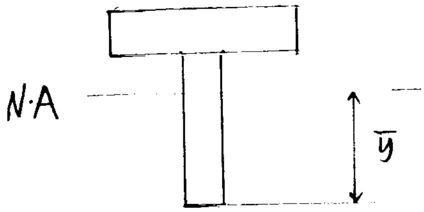
$I_z = \underbrace{2\left(\frac{1}{12} \times 500 \times 50^3 + 500 \times 50 \times \left(\frac{550}{2}\right)^2\right) + \frac{1}{12} \times 50 \times 500^3}_{\text{unit is } mm^4} = 4.32 \times 10^{-3}\ m^4$

$$q = \frac{VQ}{I_z} = \frac{5\times10^{3}\times 6.875\times10^{-3}}{4.32\times10^{-3}} = 7.96\times10^{3}\ N/m$$

$$\text{spacing} = \frac{1000}{7.96\times10^{3}} = 0.126\ m$$

Problem 3.

1.



$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{20\times200\times210 + 20\times200\times100}{20\times200 + 20\times200}$$

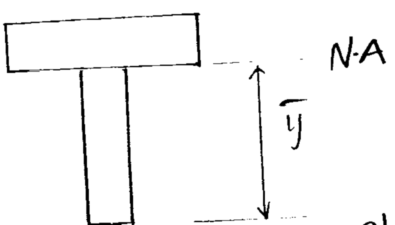
$$= 155\ mm$$

2. $$I_z = \frac{1}{12}\times200\times20^3 + 200\times20\times55^2 + \frac{1}{12}\times20\times200^3 + 200\times20\times55^2$$

$$= 3.73\times10^7\ mm^4$$

3. $$M_y = \frac{\sigma_{yp} I_z}{c} = \frac{100\times3.73\times10^7\times10^{-3}}{155} = 24.1\ kN\cdot m$$

4.



$$\left.\begin{array}{l} |\sigma_{yp}^T| = |\sigma_{yp}^C| \\ T = C \Rightarrow |\sigma_{yp}^T|\cdot A_T = |\sigma_{yp}^C|\cdot A_c \end{array}\right\} \Rightarrow A_T = A_c$$

A_T and A_c denote the area where tensile stress and compressive stress are applied, respectively

Cleary, $\bar{y} = 200$ mm

5. $$M_{ult} = T\cdot\frac{b+h}{2} = \sigma_{yp}\times b\times h\times\frac{b+h}{2} = 100\times20\times200\times\frac{220}{2}\times10^{-3}$$

$$= 44.0\ kN\cdot m$$

Problem 4

(1) $q(x) = -M\langle x-a\rangle^{-2}$

(2). 1. Boundary Conditions:

$x=0: \quad V(0) = M(0) = 0$

$x=a+b: \quad v(a+b) = v'(a+b) = 0$

2. $q(x) = EI\,v^{(4)} = -M\langle x-a\rangle^{-2}$

$V(x) = EI\,v''' = -M\langle x-a\rangle^{-1} + C_1, \quad V(0)=0 \Rightarrow C_1 = 0$

$M(x) = EI\,v'' = -M\langle x-a\rangle^{0} + C_2, \quad M(0)=0 \Rightarrow C_2 = 0$

$EI\,v' = -M\langle x-a\rangle + C_3, \quad v'(a+b)=0 \Rightarrow C_3 = Mb$

$EI\,v = -\frac{M}{2}\langle x-a\rangle^2 + Mbx + C_4, \quad v(a+b)=0 \Rightarrow C_4 = -\frac{Mb^2}{2} - Mab$

then, $v(x) = \frac{1}{EI}\left[-\frac{M}{2}\langle x-a\rangle^2 + Mbx - \frac{Mb^2}{2} - Mab\right]$

3. $x=0 \Rightarrow v(0) = -\frac{Mb^2}{2EI} - \frac{Mab}{EI}$

Section II

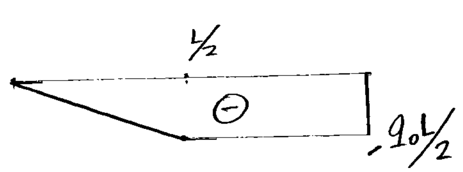
Problem 1.

(a) $q(x) = -q_0 + q_0\langle x - L/2\rangle^0$

$V(x) = \int q(x)dx = -q_0 x + q_0\langle x - L/2\rangle + C_1$

$M(x) = \int V(x)dx = -\frac{q_0}{2}x^2 + \frac{q_0}{2}\langle x - L/2\rangle^2 + C_1 x + C_2$

$V(0) = M(0) = 0 \Rightarrow C_1 = C_2 = 0$



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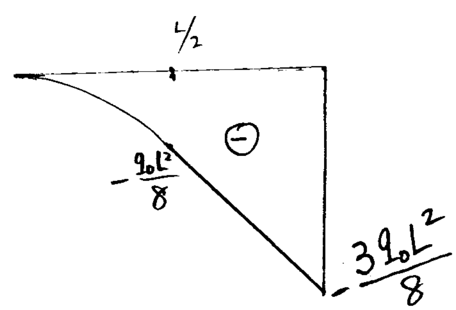


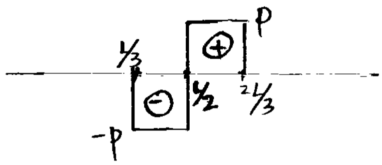
(b) $q(x) = -P\langle x - L/3 \rangle^{-1} + 2P\langle x - L/2 \rangle^{-1} - P\langle x - 2L/3 \rangle^{-1}$

$V(x) = \int q(x)\,dx = -P\langle x - L/3 \rangle^{0} + 2P\langle x - L/2 \rangle^{0} - P\langle x - 2L/3 \rangle^{0} + C_1$

$M(x) = \int V(x)\,dx = -P\langle x - L/3 \rangle + 2P\langle x - L/2 \rangle - P\langle x - 2L/3 \rangle + C_1 x + C_2$

$V(0)=0$ (No support reactions) $\Rightarrow C_1 = 0$.

$M(0)=0 \Rightarrow C_2 = 0$.



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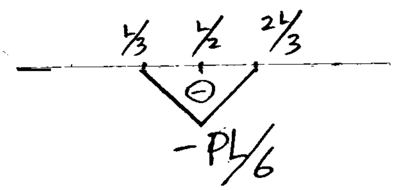
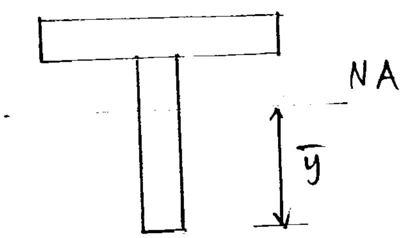


moment

Problem 2.

$$\bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{50\times 500\times 525 + 50\times 500\times 250}{50\times 500 + 50\times 500}$$

$$= 387.5 \text{ mm}$$



$$I_z = \frac{1}{12}\times 500\times 50^3 + 500\times 50\times 137.5^2 + \frac{1}{12}\times 50\times 500^3 + 500\times 50\times 137.5^2$$

$$= 9.94\times 10^8 \text{ mm}^4$$

$Q = 50\times 500\times 137.5 = 3.44\times 10^6 \text{ mm}^3$

$$q = \frac{VQ}{I_z} = \frac{5\times 10^3 \times 3.44\times 10^6}{9.94\times 10^8} \times 10^{+3} = 17.3 \text{ KN/m}$$

$$\text{spacing} = \frac{1000}{17.3\times 10^3} = 0.06 \text{ m}$$

Problem 3.

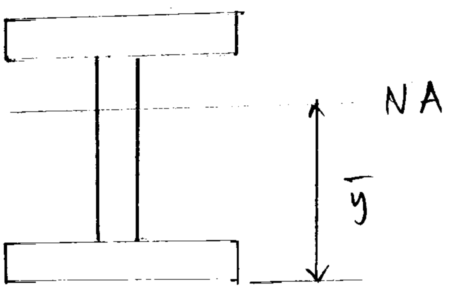
1. By symmetry, neutral axis lies in the middle. $\bar{y} = 0$

2. $I_z = \frac{1}{12} \times 20 \times 200^3 + \left(\frac{1}{12} \times 200 \times 20^3 + 200 \times 20 \times 110^2\right) \times 2$

$= 1.1 \times 10^8 \ mm^4$

3. $M_y = \frac{\sigma_{yp} I_z}{c} = \frac{50 \times 10^6 \times 1.1 \times 10^8}{120} \times 10^{-9} = 45.8 \ kN \cdot m$

4. $\left.\begin{array}{l} |\sigma_{yp}^T| = 2|\sigma_{yp}^c| \\ T = C \Rightarrow |\sigma_{yp}^T| \cdot A_T = |\sigma_{yp}^c| \cdot A_c \end{array}\right\} \Rightarrow A_T = \frac{1}{2} A_c$



Assume the upper portion is in tension and the lower portion in compression.

$20 \times 200 + (220 - \bar{y}) \times 20$

$= \frac{1}{2}\left[20 \times 200 + (\bar{y} - 20) \times 20\right]$

$\bar{y} = 220$ mm

5. $M_{ult} = 100 \times 20 \times 200 \times \frac{20}{2} + 50 \times 20 \times 200 \times \frac{200}{2} + 50 \times 20 \times 200 \times 210$

$= 66 \times 10^6 \ MPa \cdot mm^3$

$= 66 \ kN \cdot m$

Problem 4.

(1) $q(x) = -P\langle x-a \rangle^{-1}$

(2)

1. $V(0) = M(0) = 0, \quad v(a+b) = v'(a+b) = 0$

2. $q(x) = EI\,v^{(4)} = -P\langle x-a\rangle^{-1}$

$V(x) = EI\,v''' = -P\langle x-a\rangle^0 + C_1, \quad V(0) = 0 \Rightarrow C_1 = 0$

$EI\,v'' = M(x) = -P\langle x-a\rangle + C_2, \quad M(0) = 0 \Rightarrow C_2 = 0$

$EI\,v' = -\frac{P}{2}\langle x-a\rangle^2 + C_3 \quad v'(a+b) = 0 \Rightarrow C_3 = \frac{Pb^2}{2}$

$EI\,v = -\frac{P}{6}\langle x-a\rangle^3 + \frac{Pb^2}{2}x + C_4, \quad v(a+b) = 0 \Rightarrow C_4 = -\frac{Pab^2}{2} - \frac{Pb^3}{3}$

$v(x) = \frac{1}{EI}\left[-\frac{P}{6}\langle x-a\rangle^3 + \frac{Pb^2}{2}x - \frac{Pab^2}{2} - \frac{Pb^3}{3}\right]$

3. $v(0) = \frac{1}{EI}\left[-\frac{Pab^2}{2} - \frac{Pb^3}{3}\right]$