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University of California, Berkeley
Department of Civil and Environmental Engineering
CE120 / Instructor: Marios Panagiotou / Spring 2009

04/22/09

Mid-term Exam 2

Duration (2 hours and 15 minutes)

Name:

	Maximum Points	Score
Problem 1	30	30
Problem 2	30	30
Problem 3	30	30
Problem 4	20	20+
Total	110	110+

Problem 1 (30 points)

For the beam structure shown below:

$\gamma = 1$

i) Compute the reactions and draw the bending moment and shear force diagrams.

ii) Draw qualitatively the deflected shape.

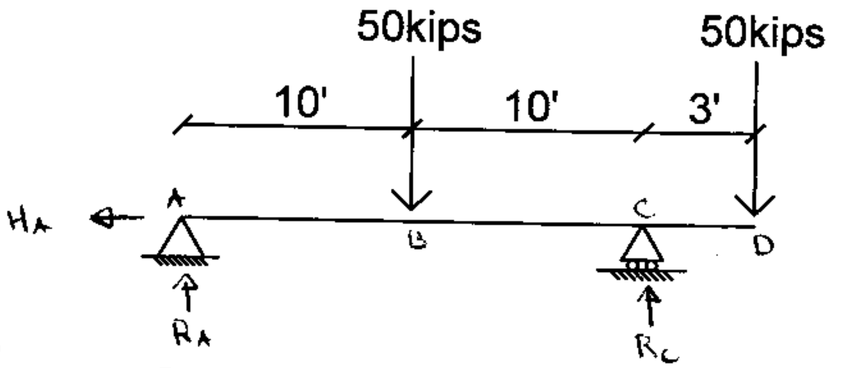
iii) Check if the beam has adequate longitudinal reinforcing steel on top. $\boxed{\phi_f = 0.9}$

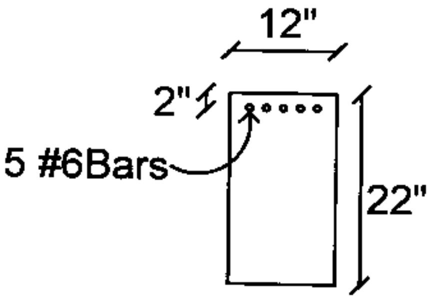
iv) Design the bottom longitudinal reinforcing steel as well as the reinforcing steel for shear. Show clear sketches of the side view and section view of your design. $\boxed{\phi_s = 0.7}$

Notes:

1) For the design part you have to use the **LRFD** method. **Do not** consider any load factors.

2) For concrete it is given $f_c' = 4$ ksi and for steel $f_y = 60$ ksi.





Beam Section

i)

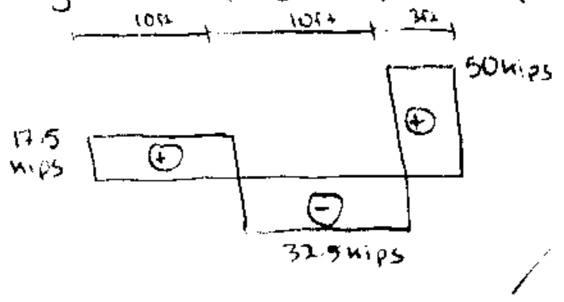
$\Sigma F_x = 0 \Rightarrow \boxed{H_A = 0}$

$\Sigma M_A = 0 \Rightarrow -50\text{kips}(10\text{ft}) + R_C(20\text{ft}) - 50\text{kips}(23\text{ft}) = 0$

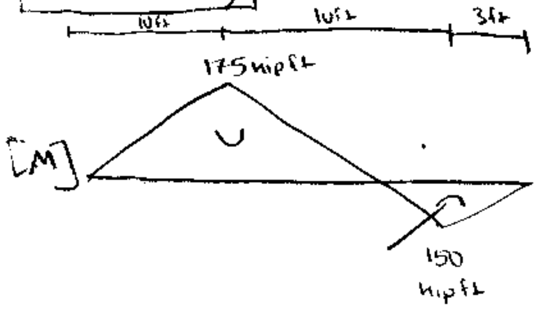
$\Rightarrow \boxed{R_C = 82.5 \text{ kips}}$

$M = -17.5x + 50(x-10) - 82.5(x-20)$

$\Sigma F_y = 0 \Rightarrow R_A + 82.5\text{kips} - 100\text{kips} = 0 \Rightarrow \boxed{R_A = 17.5 \text{ kips}}$

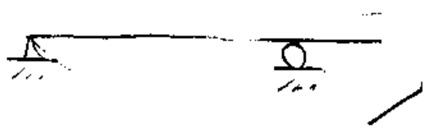


[V]

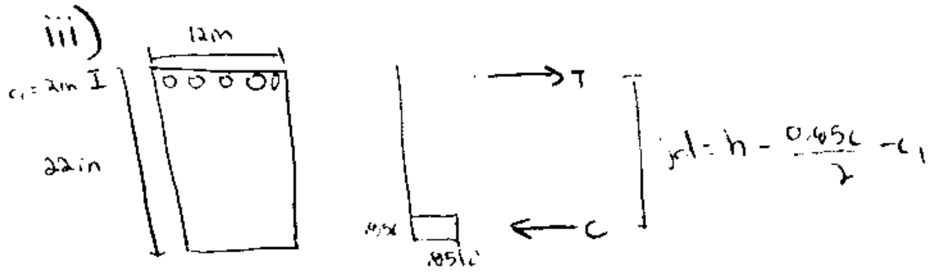


not to scale

ii)



iii)



$A_{sT} = 5 \cdot \frac{\pi}{4}\left(\frac{6}{8}in\right)^2$
$= 2.209 in^2$

$|M_U^-| = 150 kipft$

$T = A_{sT} f_y$

$C = (0.85)^2 f_c' cb$

$T = C \Rightarrow c = \frac{A_{sT} f_y}{(0.85)^2 b f_c'} = \frac{(2.209 in^2)(60ksi)}{(0.85)^2 (12in)(4ksi)} = 3.822 in$

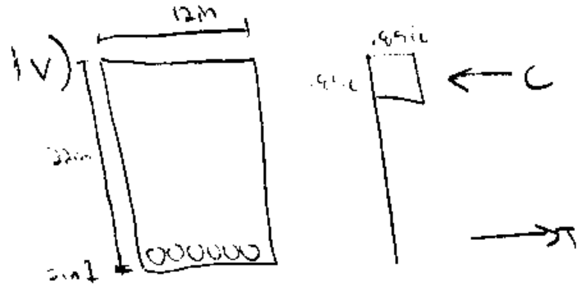
$\sum M_c = M_n = T(jd) = A_{sT} f_y \left(h - \frac{0.85c}{2} - c_1\right) = (2.209 in^2)(60ksi)\left(22in - \frac{0.85(3.822in)}{2} - 2in\right)$

$M_n = 2435.509 \text{ kip in}$

$\frac{M_U}{\varnothing_f} = \frac{150 kipft}{0.9} \times \frac{12in}{1ft} = 2000 \text{ kip in}$

check if $M_n \geq \frac{M_U}{\varnothing_f} \Rightarrow 2435.51 \text{ kipin} \geq 2000 \text{ kipin}$ ✓ yes, adequate reinforcing steel on top

iv)



Same eqns as above

$c = \frac{A_{sb} f_y}{(0.85)^2 b f_c'}$

$|M_U^+| = 175 kipft$

$M_n = A_{sT} f_y \left(h - \frac{A_{sT} f_y}{2(0.85)^2 b f_c'} - c_1\right) \geq \frac{M_U}{\varnothing_f}$

$-A_{sT}^2 \left(\frac{f_y^2}{2(0.85)^2 b f_c'}\right) + A_{sT}\left(f_y(h - c_1)\right) - \frac{M_U}{\varnothing_f} = 0$

$-A_{sT}^2 \left(\frac{(60ksi)^2}{2(0.85)^2 (12in)(4ksi)}\right) + A_{sT}\left(60ksi(22in - 2in)\right) - \frac{175 kipft}{0.9} \times \frac{12in}{ft} = 0$

$-A_{sT}^2 (44.1176 \text{ kip}/in^3) + A_{sT}(1200 \text{ kip}/in) - 2333.3333 \text{ kipin} = 0$

$A_{sT} = 2.1078 in^2$

area of #6 bar $= 0.44 in^2 \rightarrow$ $\boxed{\text{need 5 \#6 bars}}$ $(A_{sT} = 2.21 in^2)$

$s_{max} = \frac{d}{2} = \frac{h-c_1}{2} = \frac{22in - 2in}{2} = 10in$ $\qquad |V_u| = 50kips$

try $\boxed{\#4\ bar}$

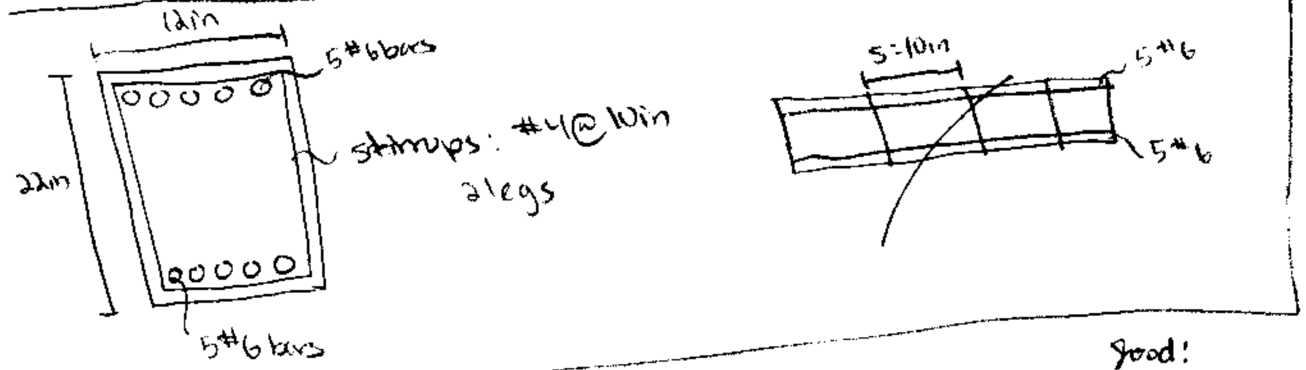
$V_c = 2\frac{\sqrt{f'_c}}{1000} bd = 2\frac{\sqrt{4000}}{1000}ksi\,(12in)(20in) = 30.358\ kips$

$V_s = A_v f_y \frac{d}{s} = 2\times\frac{\pi}{4}\left(\frac{4}{8}in\right)^2 (60ksi)\left(\frac{20in}{s}\right) = \frac{471.239\ kip\ in}{s}$

$V_n = V_c + V_s \geq \frac{V_u}{\phi_s} = \frac{50kips}{0.7} = 71.429kips$

$30.358kip + \frac{471.239\ kip\,in}{s} \geq 71.429kips \Rightarrow s = 11.47in$

since $s > s_{max}$, choose $\boxed{S_{max} = 10in}$



good!

$\boxed{\begin{array}{l}\phi_f = 0.9\\ \phi_s = 0.7\end{array}}$

Problem 2 (30 points)

For the frame structure shown below:

i) Compute the reactions and draw the bending moment, shear force and axial force diagrams.

ii) Find the dimension b_f of the I-section in order members BC and CD to have adequate flexural and shear strength.

iii) Find the minimum section moment of inertia I_{AB} in order member AB to have adequate strength against buckling. The critical buckling load is $P_{cr} = \pi^2 EI / (\kappa L)^2$, κ=1.

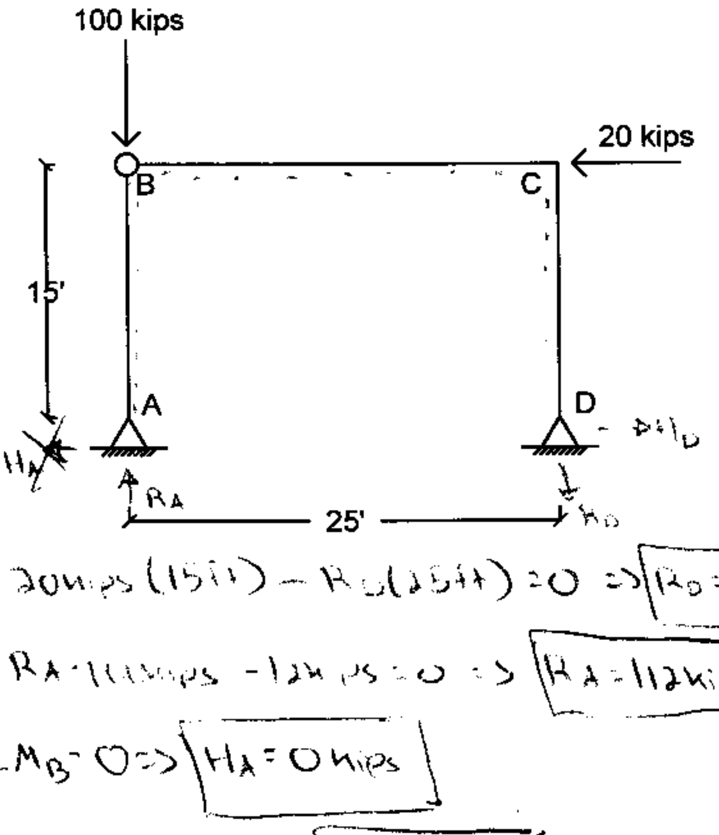
Notes:

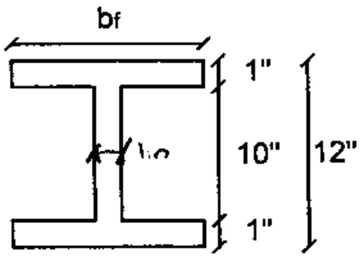
1) For the design part you have to use the **LRFD** method. **Do not** consider any load factors.

2) Find only one b_f which is adequate for bending and shear for both members BC and CD.

3) Consider A36 steel, E=29000 ksi.

$f_y = 36ksi$





Beam Section

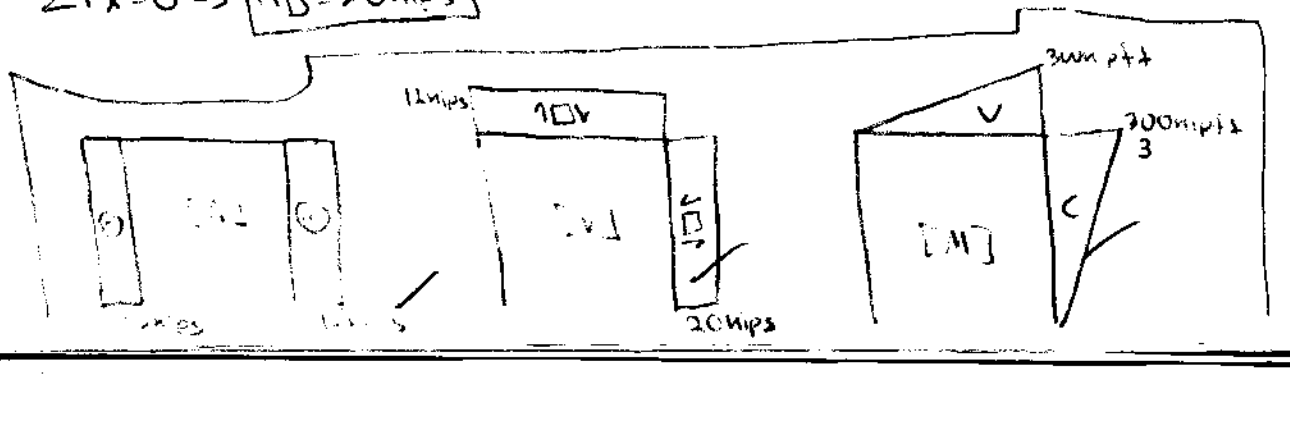
1)

$\Sigma M_A = 0 \Rightarrow 20kips(15ft) - R_D(25ft) = 0 \Rightarrow \boxed{R_D = 12kips}$

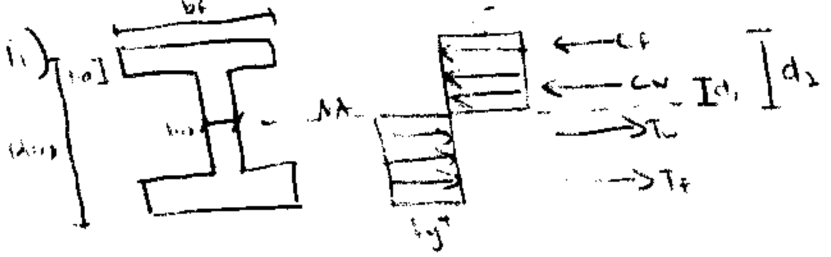
$\Sigma F_y = 0 \Rightarrow R_A - 100kips - 12kips = 0 \Rightarrow \boxed{R_A = 112kips}$

$\Sigma M_B = 0 \Rightarrow \boxed{H_A = 0kips}$

$\Sigma F_x = 0 \Rightarrow \boxed{H_D = 20kips}$



i)



$d_1 = \frac{1}{2}\left(\frac{h}{2} - t_f\right)$

$d_2 = \frac{h}{2} - \frac{t_f}{2} = \frac{1}{2}(h - t_f)$

$|M_u^+| = |M_u^-| = 300\,\text{kipft}$

$\Sigma M_{NA} = 2C_w d_1 + 2C_f d_2$

$= 2\left(f_y t_w\left(\frac{h}{2} - t_f\right)\right)\left(\frac{1}{2}\left(\frac{h}{2} - t_f\right)\right) + 2\left(f_y b_f t_f\right)\left(\frac{1}{2}(h-t_f\right.$

$C_f = f_y b_f t_f = T_f$

$C_w = f_y t_w\left(\frac{h}{2} - t_f\right)$

$\Sigma M_{NA} = f_y t_w\left(\frac{h}{2} - t_f\right)^2 + \left(f_y b_f t_f (h - t_f)\right)$

$= (36\text{ksi})(1\text{in})(6\text{in} - 1\text{in})^2 + b_f(36\text{ksi})(1\text{in})(12\text{in} - 1\text{in})$

$= 900\,\text{kipsin} + (b_f)396\,\text{kip} = M_n \geq \frac{M_u}{\phi_f} = \frac{300\,\text{kipft}}{0.9} \times \frac{12\text{in}}{1\text{ft}} = 4000\,\text{kip}$

$\Rightarrow \boxed{b_f^{min} = 7.83\,\text{in}}$ for bending

$|V_u| = 20\,\text{kips}$

shear: only web carries shear $V_n = A_v f_v = \frac{h t_w f_y}{\sqrt{3}} = \frac{(12\text{in})(1\text{in})(36\text{ksi})}{\sqrt{3}} = 249.42\,\text{kips}$

$V_n \geq \frac{V_u}{\phi_s} = \frac{20\,\text{kips}}{0.7} = 28.57\,\text{kips}$

$249.42\,\text{kips} \geq 28.57\,\text{kips}$ ✓

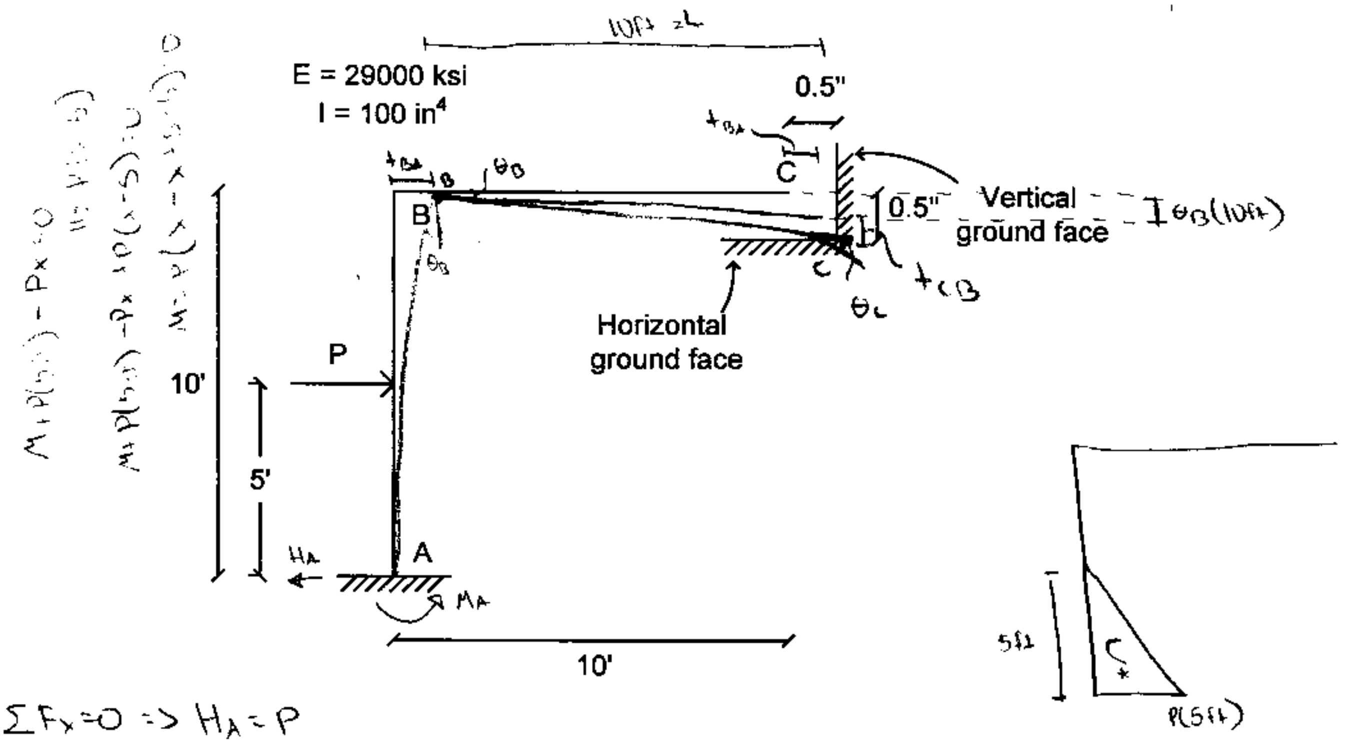
bf isn't taken into account when finding adequate shear strength

ii) buckling: $P_{cr} = \frac{\pi^2 EI}{(kL_{AB})^2} = \frac{\pi^2(29000\,\text{ksi})\,I_{AB}}{\left(15\text{ft} \times \frac{12\text{in}}{\text{ft}}\right)^2} >> |N_{max}| = 112\,\text{kips}$

$\Rightarrow \boxed{I_{AB}^{min} = 12.68\,\text{in}^4}$

Problem 3 (30 points)

For the structure shown below find the value of force P for which point C will touch the ground. Which face (vertical or horizontal) of the ground point C will touch? Members AB, BC have the same EI shown below.



$\Sigma F_x = 0 \Rightarrow H_A = P$

$\Sigma M_A = 0 \Rightarrow M_A - P(5ft) = 0 \Rightarrow M_A = P(5ft)$

$\Delta C_x = t_{BA}$

$\Delta C_y = \theta_B L + t_{CB}$

$$\theta_A = 0 \Rightarrow \Delta\theta_{AB} = \theta_B = \int_0^L \phi(x)\,dx = \int_0^L \frac{M(x)}{EI}\,dx = \frac{P(5ft)}{EI} \cdot \frac{5ft}{2} \times \left(\frac{12in}{1ft}\right)^2 = \frac{1800\,P\,in^2}{EI}$$

$$t_{BA} = \int_A^B \phi(x)\,x\,dx = \int_A^B \frac{M(x)}{EI}\,x\,dx = \frac{P(5ft)}{EI} \times \frac{5ft}{2} \times \left(10ft - \frac{1}{3}5ft\right) \times \left(\frac{12in}{1ft}\right)^3 = \frac{1.8(10^5)\,P\,in^3}{EI}$$

$$t_{CB} = \int_B^C \frac{M(x)}{EI}\,x\,dx = 0 \leftarrow \text{no bending in member BC}$$

$$\Delta C_x = \frac{1.8(10^5)\,P\,in^3}{EI}$$

$$\Delta C_y = \frac{1800\,P\,in^2}{EI} \times 10ft \times \frac{12in}{1ft} = \frac{2.16(10^5)\,P\,in^3}{EI}$$

(see other side for answer)

$$\Delta C_x = 0.5\,in = \frac{1.8(10^5)\,in^3 \cdot P}{(29000\,ksi)(100\,in^4)} \Rightarrow P = 8.056\,kips$$

$$\Delta C_y = 0.5\,in = \frac{2.16(10^5)\,in^3\,P}{(29000\,ksi)(100\,in^4)} \Rightarrow \boxed{P = 6.713\,kips}$$

$\boxed{\text{point C will touch horizontal ground face}}$

also because for same value of P, $\Delta C_y > \Delta C_x$

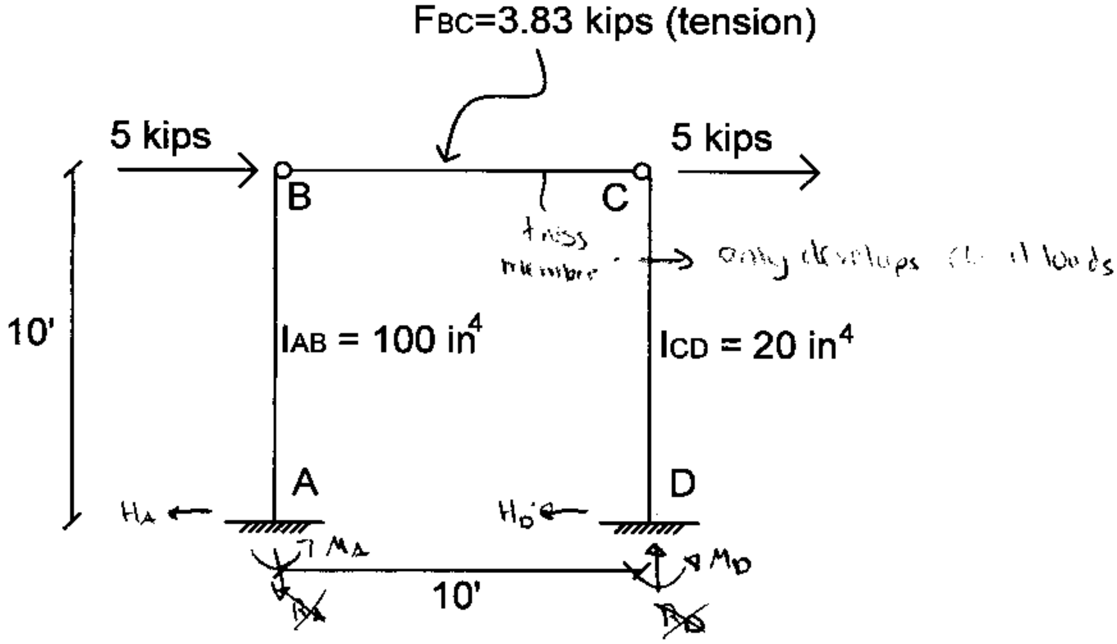
↳ more displacement in y direction

Problem 4 (20 points)

The structure shown below is indeterminate to the first degree. It is given that the developed force in member BC is F_{BC} = 3.83 kips and is ***tension***. What is the section area A_{BC} of member BC? For all members E=29000 ksi.

equilibrium
displacements

3.32



E=29000 ksi
(all the members)

5kips → B → 3.83kips

HA ← MA

RA

$\Sigma F_x = 0$

$\Rightarrow H_A = 8.83\,\text{kips}$

$\Sigma M_A = 0 \Rightarrow M_A - 8.83\,\text{kips}(10\,\text{ft}) = 0$

$\Rightarrow M_A = 88.3\,\text{kipft}$

3.83kips ← C → 5kips

HD ← D MD

RD

$\Sigma F_x = 0$

$\Rightarrow H_D = 1.17\,\text{kips}$

$\Sigma M_D = 0 \Rightarrow M_D - 1.17\,\text{kips}(10\,\text{ft}) = 0$

$\Rightarrow M_D = 11.7\,\text{kipft}$

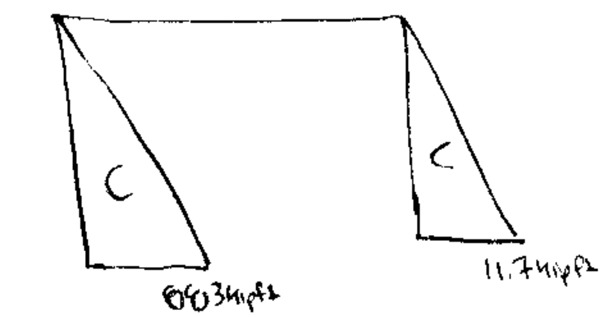
global equilibrium

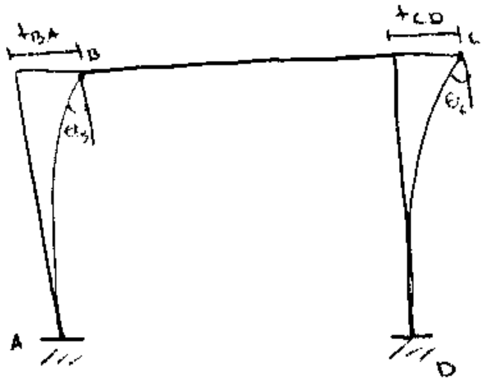
$\Sigma M_A = 0 \Rightarrow 88.3\,\text{kipft} - 5\,\text{kip}(10\,\text{ft}) - 5\,\text{kip}(10\,\text{ft}) + 11.7\,\text{kipft} - R_D(10\,\text{ft}) = 0$

$\Rightarrow R_D = 0$

$\Sigma F_y = 0 \Rightarrow R_A = 0$

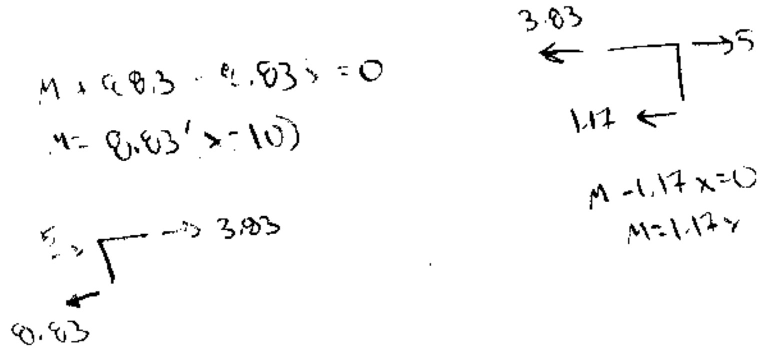
[M]





$M + 88.3 - 8.83x = 0$

$M = 8.83(x - 10)$



$M - 1.17x = 0$

$M = 1.17x$

if $t_{BA} = t_{CD}$, member BC does not bend at joints

$$t_{BA} = \int_A^B \frac{M(x)}{EI} x\,dx = \frac{88.3\text{kipft}}{EI} \times \frac{10\text{ft}}{2} \times \frac{2}{3} \times 10\text{ft} = \frac{2943.33\text{kipft}^3}{EI_{AB}}$$

$$t_{CD} = \int_C^D \frac{M(x)}{EI} x\,dx = \frac{11.7\text{kipft}}{EI} \times \frac{10\text{ft}}{2} \times \frac{2}{3} \times 10\text{ft} = \frac{390\text{kipft}^3}{EI_{CD}}$$

$t_{CD} < t_{BA}$

$$\Rightarrow |\Delta L| = |t_{BA} - t_{CD}| = \frac{1}{29000\text{ksi}}\left[\frac{2943.33\text{kipft}^3}{160\text{in}^4} - \frac{390\text{kipft}^3}{20\text{in}^4}\right]\left(\frac{12\text{in}}{1\text{ft}}\right)^3 = 0.592\text{in}$$

$$\frac{\Delta L}{L} = \varepsilon = \frac{0.592\text{in}}{10\text{ft}} = 0.0049\text{in} \quad \frac{\sigma}{E} = \frac{F_{BC}}{A_{BC}E} \Rightarrow A_{BC} = \frac{F_{BC}}{\varepsilon E} = \frac{3.83\text{kips}}{0.0049(29000\text{ksi})}$$

$$= \boxed{0.0268\text{ in}^2}$$

~~member BC is in tension, but right there BC is acting... compressed or ...~~

excellent

you are better than the Professor! :)