

MIDTERM EXAM 1

Name and Student ID: _____

SOLUTION

Instructions: Answer the questions that follow directly on these pages in the spaces provided. Use the back of the page if you need more room for your answer. If you believe there is insufficient information provided to answer a question completely, state reasonable additional assumptions and proceed from there.

This exam is closed-book/closed-notes. Calculators are not allowed.

Time: 50 minutes.

Question:	Score:	Out of:
1	_____	5
2	_____	5
3	_____	4
4	_____	5
BONUS	_____	
TOTAL	_____	19

Useful Data and Formulas:

Ideal gas law: $PV = nRT$

Hydrostatic equation: $dP/dz = -\rho g$

CMFR general equation: $dC/dt = S - LC$

Solution to CMFR general equation: $C(t) = C_0 \exp(-Lt) + (S/L)[1 - \exp(-Lt)]$

Exponential growth equation: $dN/dt = rN$

Logistic growth equation: $\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right)$

1. AIR AND WATER BASICS (1 point each)

a) The value of the ideal gas constant depends on its units. Write one valid set of units for the ideal gas constant (just the units, not the value).

$$PV = nRT$$

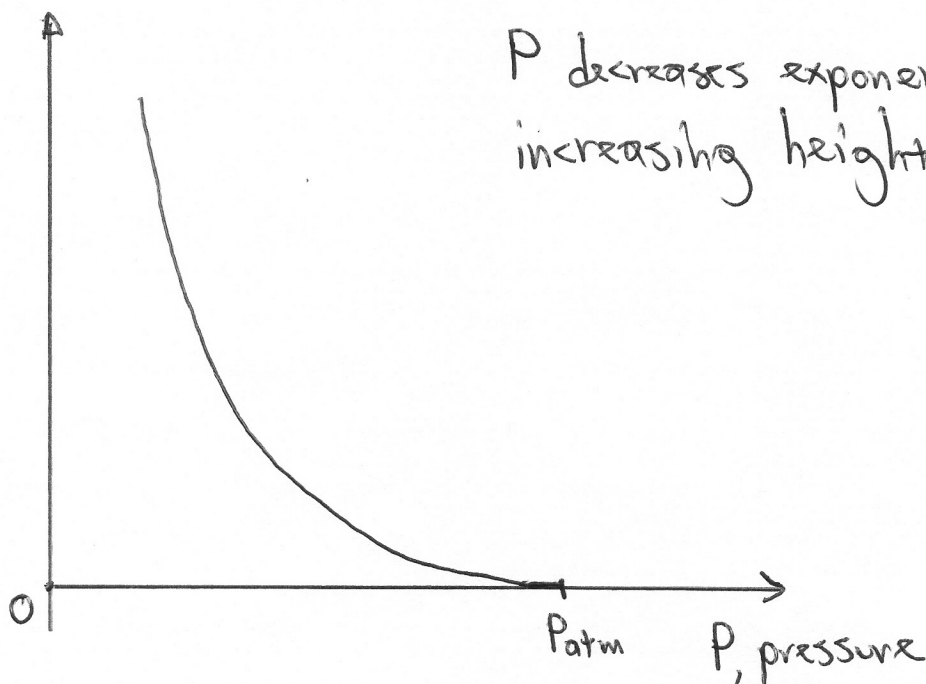
$$R = \frac{PV}{nT} [=] \frac{\text{atm} \cdot \text{m}^3}{\text{mol} \cdot \text{K}} \quad \text{or} \quad \frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \text{K}} \quad \text{or} \quad \frac{\text{N} \cdot \text{m}}{\text{mol} \cdot \text{K}} \quad \text{or} \quad \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

b) What are the names of the two layers of the atmosphere nearest the ground, and what is fundamentally different within these layers that distinguish them from each other?

Troposphere : temperature decreases w/ altitude

Stratosphere : temperature increases w/ altitude

c) Sketch a plot of atmospheric pressure as a function of altitude and identify the point of maximum pressure. State how pressure varies with altitude.



1. AIR AND WATER BASICS – continued

d) Given the follow components of the hydrologic cycle, calculate the characteristic lifetime of a water molecule in the ocean.

Depth of ocean = 10^3 m

Volume of ocean = 10^{18} m³

Precipitation onto ocean = 4×10^{14} m³ yr⁻¹

Runoff into ocean = 10^{14} m³ yr⁻¹

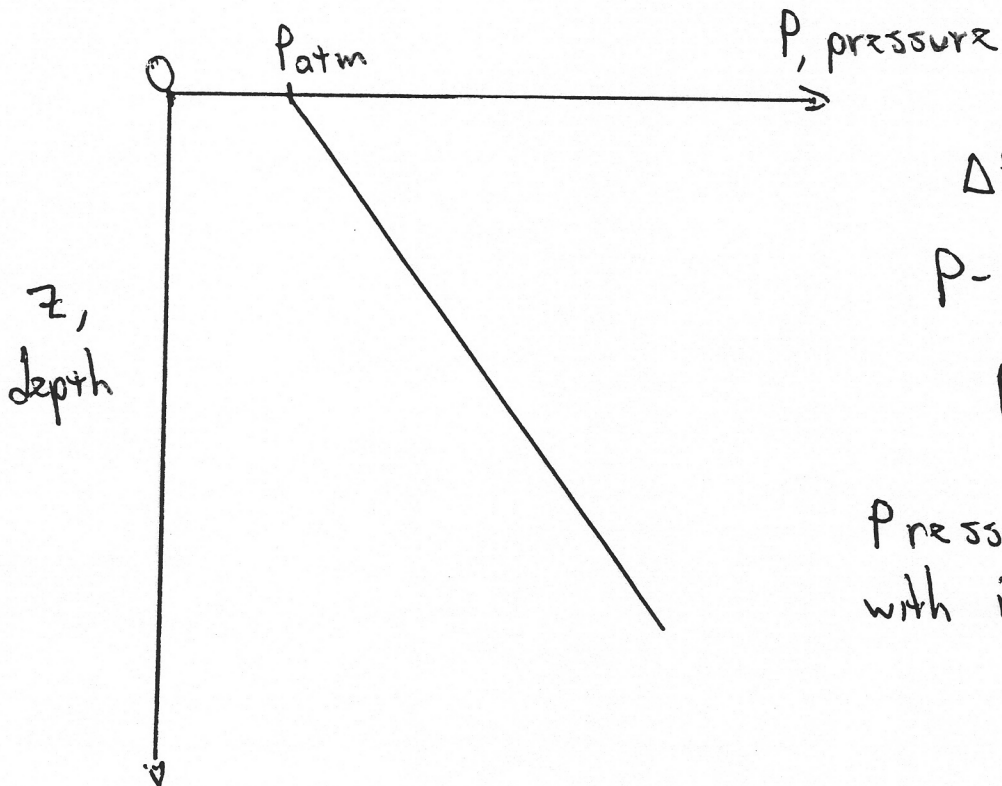
Evaporation from ocean = 5×10^{14} m³ yr⁻¹

$$\tau = \text{stock} / \text{flow}$$

$$\text{flow} = \text{evaporation} = (\text{precipitation} + \text{runoff})$$

$$\tau = \frac{10^{18} \text{ m}^3}{5 \times 10^{14} \text{ m}^3/\text{yr}} = \frac{1}{5} \times 10^4 \text{ yr} = 2000 \text{ yr}$$

e) Assume that the density of water is constant. Sketch a plot of pressure as a function of depth in the ocean and identify the point of minimum pressure. State how pressure varies with depth.



$$\Delta P = \rho g \Delta z$$

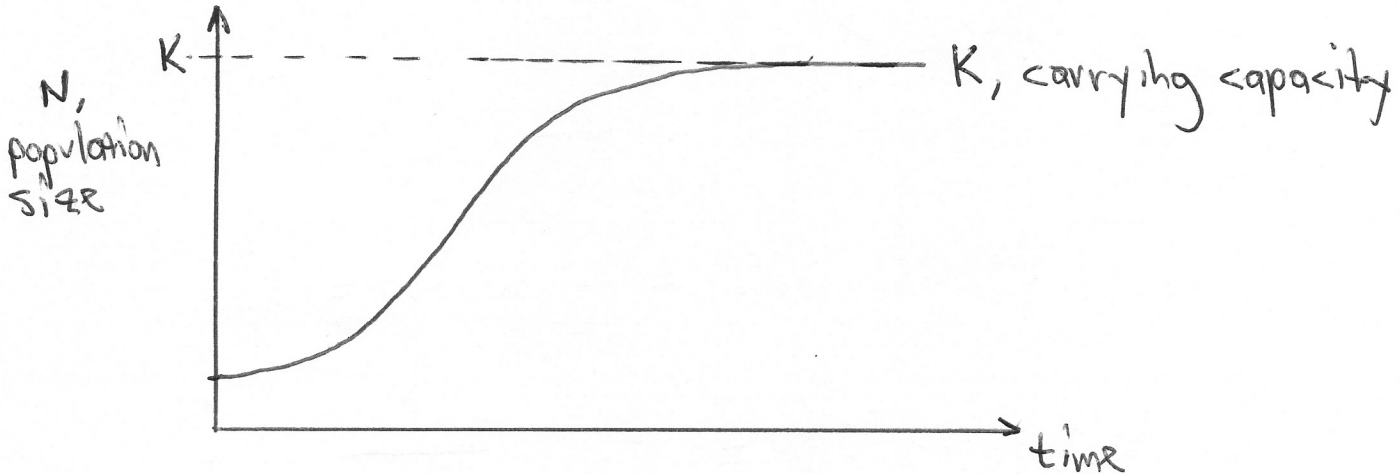
$$P - P_{\text{atm}} = \rho g (z - \phi)$$

$$P = P_{\text{atm}} + \rho g z$$

Pressure increases linearly with increasing depth.

2. POPULATION GROWTH (1 point each)

a) Sketch a logistic growth curve and label the axes. Define K and identify it on the curve.



b) What is the population size when dN/dt attains its maximum value?

$$K/2$$

c) Given the governing equation for logistic growth on the front page of the exam, derive an expression for the instantaneous growth rate, R .

$$R = \frac{dN/dt}{N} = r \left(1 - \frac{N}{K} \right)$$

2 ~~3~~ POPULATION GROWTH – continued

d) What are the first three phases of demographic transition?

1. High birth and high death rates \Rightarrow slow population growth
2. Death rates decrease \Rightarrow growth rate increases (natural increase)
3. Birth rates decrease \Rightarrow growth slows

e) Assuming an annual exponential growth rate of 35%, about how many years will it take for a population to increase from 100 to 400?

From 100 to 400 is two doublings in population size

100
200
400

Doubling time, $T_d \approx \frac{70}{r\%}$

$$T_d = \frac{70}{35\% \text{yr}^{-1}} = 2 \text{ yr for one doubling}$$

So, two doublings takes 4 yrs.

3. HOT SOUP (4 points)

I prepared a bowl of tomato soup (mass = m_s). It was too hot to eat, and it was in a well insulated bowl, so I knew it wasn't going to cool anytime soon. I decided to put an ice cube (mass = m_i) into it to cool it. As I was stirring the ice cube into the soup and watching it melt, I wondered how much the soup would cool.

If the initial temperature of the ice is $T_i = 0^\circ\text{C}$, and the initial temperature of the soup is T_s , derive an expression for the final temperature of the soup, T_f .

Assume that the specific heat of the soup, c , is equal to that of water, and assume that the specific heat of water is constant in the range from 0 to 100°C .

Define all symbols in your equations unless they are explicitly defined above.

Energy balance : heat lost by soup = heat gained by ice

$$m_s c \Delta T_s = m_i \lambda + m_i c \Delta T_i$$

sensible heat
term for soup

sensible heat and latent heat terms
① ice melts, (latent heat), ② melted ice water warms to final temp. (sensible heat)

- λ = latent heat of fusion (melting)
- ΔT_s = change in T of soup = $T_s - T_f$
- ΔT_i = change in T of ice = $T_f - T_i = T_f - 0^\circ\text{C} = T_f$
after it melts

Substituting yields : $m_s c T_s - m_s c T_f = m_i \lambda + m_i c T_f$

$$c(m_i + m_s) T_f = m_s c T_s - m_i \lambda$$

$$T_f = \frac{m_s c T_s - m_i \lambda}{c(m_i + m_s)}$$

4.5. HOLY SMOKES – MATERIALS BALANCE (5 points)

It is raining outside and your friend insists that you have a barbeque inside your single room apartment shaped like a box. Barbequing indoors is very dangerous, as you are well aware. To prove it to your friend, you calculate the concentration (C , g m^{-3}) of carbon monoxide gas (CO) in your apartment that would result from such a stunt. You make reasonable assumptions for the barbecue CO emission factor (E , g s^{-1}) and your apartment ventilation rate (Q , $\text{m}^3 \text{s}^{-1}$) and volume (V , m^3).

- a) Write the appropriate materials balance equation for this system. Do not assume steady state, but state all other assumptions that you make.

accumulation = inflow + emission - outflow - decay
 assume inflow = ϕ (fresh air) and decay = ϕ (conservative pollutant)

$$V \frac{dC}{dt} = E - QC$$

- b) Write an expression for the characteristic time for this system to reach steady state.

For $\frac{dC}{dt} = S - LC$, $\hat{\tau} = \frac{1}{L}$

Here, $\hat{\tau} = \frac{V}{Q}$

- c) Write an expression for the steady state CO concentration.

Steady state means $\frac{dC}{dt} = 0$

$\therefore E = QC_{ss}$

$$C_{ss} = E/Q$$

47. HOLY SMOKES – MATERIALS BALANCE – continued

d) You decide to have the barbeque in your apartment while you wait outside in the rain. The barbecue lasts several times longer than the characteristic time of the system. To determine when it is safe to go back inside, you calculate the CO concentration as a function of time, $C(t)$, after the barbeque is extinguished. Derive an expression for $C(t)$ that is valid after the barbeque is extinguished. Define any new parameters (i.e., symbols) that you introduce.

When bbq is extinguished, $E = \phi$

Mass balance eqn is $\frac{dc}{dt} = -\frac{Q}{V} C$

$$\int_{C_0}^C \frac{dc}{c} = -\frac{Q}{V} \int_0^t dt$$

$$\therefore C(t) = C_0 \exp\left(-\frac{Q}{V} t\right)$$

where t is time since bbq. was extinguished (i.e., since $t=0$) and C_0 is the S.S. concentration determined in part c.

$$\Rightarrow \boxed{C(t) = \frac{E}{Q} \exp\left(-\frac{Q}{V} t\right)}$$

BONUS QUESTIONS (0.5 point each)

In the movie "The Next Industrial Revolution," what did waste equal?

Waste = Food

The Kyoto Protocol was finally positioned to enter into force when it was ratified by which country?

Russia