

Midterm 1: Tension and Compression Members

10/14/08, 502 Davis Hall, 2 hours

Name _____

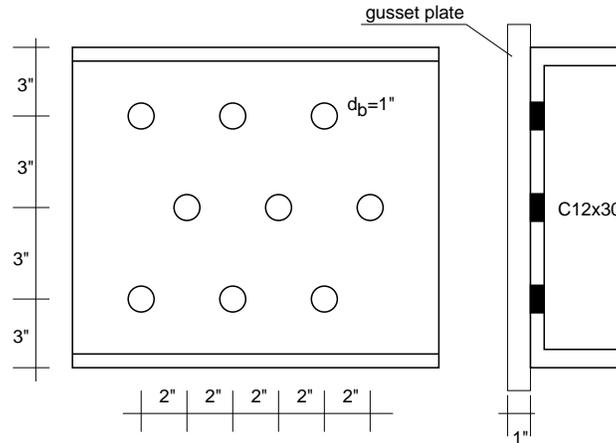
Problem	Points	Maximum
1		25
2		25
3		25
4		25
total		100

Honor Pledge:

I have neither give nor received aid during this examination, nor have I concealed any violation of the Honor Code.

Problem 1: (25%)

Determine the governing effective area ($A_e = U A_n$) for the C12x30 channel shown (not the gusset plate). Standard holes are made for 1-inch diameter bolts by punching. Compute the appropriate shear lag factor and include it in your calculation. Do not consider block shear.



Problem 2: (25%)

A W24x76 A992 tension member is connected at its two ends to gusset plates as shown. Standard holes are made for 7/8-inch diameter bolts by punching. Compute the design strength ϕR_n of this member taking into account yielding, ultimate tension, and block shear limit states at **both ends** of the member. To compute the shear lag factor assume the following: on the left end, gusset plates are connected to the top and the bottom flange; on the right end, two gusset plates are connected on either side of the web (as shown in Figure C-D3.1 in the commentary of the AISC Manual).



Problem 3: (25%)

An 15-foot tall W14x??? A992 steel column is a part of a frame structure. It carries a factored load of $P_u = 2008$ kips. Buckling in the plane of the frame occurs about strong axis, while buckling out-of-plane occurs about the weak axis. The effective length factor K_y for the weak axis is equal to 1.0.

1. Select a W-section for this column, assuming that the frame is not braced (sway may happen). The effective length factor K_x for the strong axis is equal to 1.9.
2. Select a W-section for this column, assuming that the frame is braced (sway can not happen). The effective length factor K_x for the strong axis is equal to 0.76.

Problem 4: (25%)

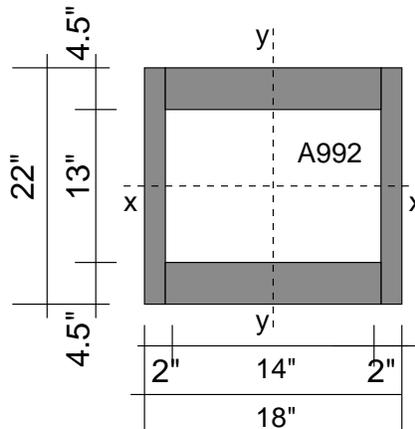
Effective lengths of the column are: $(KL)_x = 30$ feet and $(KL)_y = 22$ feet.

1. Determine the design strength (ϕP_n) of the built-up A992 steel column whose section is shown below using the AISC LRFD provisions. Check if the section is compact.
2. Determine the AISC LRFD design strength of the A992 W14x730 column section. Is this cross section is compact?
3. Compare the results. The sections have roughly the same area: why is one stronger than the other?

Reminder: properties of a built up cross-section can be computed using the parallel axis theorem:

$$\begin{aligned} A &= \sum A_{plates} \\ I_x &= I_{plates,x} + \sum A_{plates,x} d_{plates,x}^2 \\ I_y &= I_{plates,y} + \sum A_{plates,y} d_{plates,y}^2 \end{aligned}$$

where d_{plates} is the distance from the centroid of the plate area to the centroid of the cross section.



2008 10/14

Mittlerm

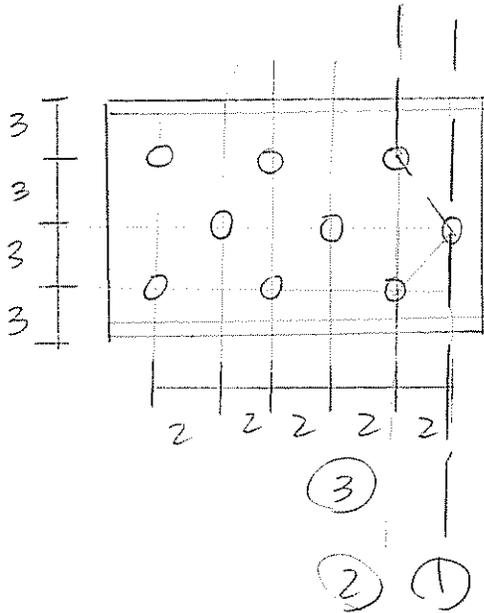
Problem 1

C12 x 30

$$A_g = 8.81 \text{ in}^2 \quad t_w = 0.510 \text{ in} \quad \bar{x} = 0.674 \text{ in}$$

 $d_b = 1 \text{ in}$

$$d_h = 1 \text{ in} + 1/8 \text{ in} = 9/8 \text{ in}$$



$$\text{Case 1: } A_n = A_g - 1(d_h)(t_w)$$

$$A_n = 8.81 - (9/8)(0.51)$$

$$A_n = 8.24 \text{ in}^2$$

$$\text{Case 2: } A_n = A_g - 3(d_h)(t_w) + 2\left(\frac{s^2}{4g}\right)t_w$$

$$A_n = 8.81 - 3(9/8)(0.51) + 2\left(\frac{2^2}{4(3.0)}\right)(0.510)$$

$$A_n = 8.81 - 1.72 + 0.34$$

$$A_n = \underline{\underline{7.43 \text{ in}^2}} \quad \underline{\underline{\text{Case 2}}}$$

Case 3:

$$A_n = A_g - 2(d_h)(t_w)$$

$$= 8.81 - 2(9/8)(0.510) = 7.66 \text{ in}^2$$

$$A_n = \frac{9}{8} [A_n] = \frac{9}{8} (7.66) = 8.62 \text{ in}^2$$

$$u = 1 - \frac{\bar{x}}{L} = 1 - \frac{0.674}{10} = 0.933$$

$$A_e = uA_n = 0.93 (8.62) = 8.01 \text{ in}^2$$

$A_e = 8.01$ $u = 0.933$

Problem 2

$W24 \times 76$
 $A_g = 22.4 \text{ in}^2$
 $t_f = 0.68 \text{ in}$
 $t_w = 0.44 \text{ in}$
 $b_f = 8.99 \text{ in}$

A992
 $F_y = 50 \text{ ksi}$
 $F_u = 65 \text{ ksi}$

$d_b = 7/8''$
 $d_n = 7/8 + 1/8 = 1''$

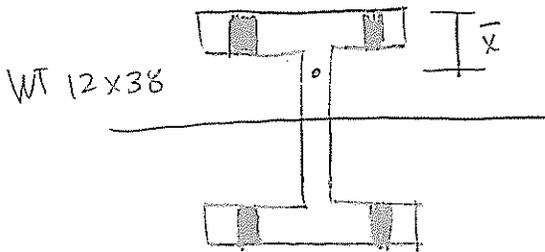
yield $\phi R_n = 0.9 F_y A_g = 0.9(50)(22.4) = \underline{1008 \text{ kips}}$

$\phi R_n = \underline{1008 \text{ kips}}$

fracture $\phi R_n = 0.75 F_u A_e$

left side : $A_n = A_g - 4(d_n)(t_f)$
 $= 22.4 - 4(1)(0.68) = 19.68 \text{ in}^2$

u



From Part 1 of manual

$\bar{x} = 3.0 \text{ in}$

$u = 1 - \frac{\bar{x}}{l} = 1 - \frac{3}{6} = 0.5$

$u = 0.5$

$A_n = 19.68 \text{ in}^2$

$A_e = u A_n = 0.5(19.68) = 9.84 \text{ in}^2$

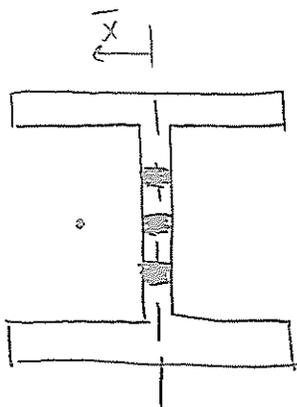
$A_e = 9.84 \text{ in}^2$

Right side : $A_n = A_g - 3(d_n)(t_w)$

$= 22.4 - 3(1)(0.44)$

$= 21.08 \text{ in}^2$

$A_n = 21.08 \text{ in}^2$



$b_f = 8.99''$

$t_f = 0.68$

$b_f > \frac{2}{3} t_f$

Table D3.1

Case 7

$u = 0.70$

$$A_e = U A_n = 0.7(21.08) = 14.76 \text{ in}^2$$

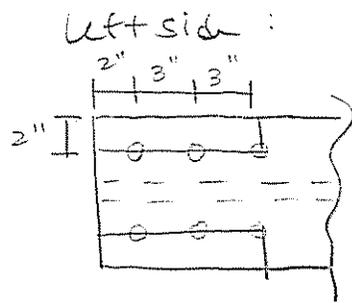
$$A_e = 14.76 \text{ in}^2$$

Left side $\phi R_n = 0.75(65)(9.86) = 480.7 \text{ kips}$

Right side $\phi R_n = 0.75(65)(14.76) = 719.55 \text{ kips}$

$$\phi R_n = 719.55 \text{ kips}$$

Block Shear



plan view

$$A_{gv} = 4(8)t_f = 4(8)(0.68) = 21.76 \text{ in}^2$$

$$A_{nv} = A_{gv} - 4(2.5)(d_h)(t_f) = 21.76 - 6.8 = 14.96 \text{ in}^2$$

$$A_{gt} = 4(2)t_f = 5.44 \text{ in}^2$$

$$A_{nt} = A_{gt} - 4\left(\frac{1}{2}\right)(d_h)(t_f) = 4.08 \text{ in}^2$$

$$R_n = 0.6 F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.6 F_y A_{gv} + U_{bs} F_u A_{nt}$$

$$R_n = 0.6(65)(14.96) + 65(4.08) \leq 0.6(50)(21.76) + 65(4.08)$$

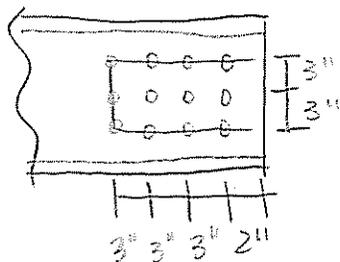
$$R_n = 848.64 \leq 918$$

← governs

$$\phi R_n = 0.75(848.64) = 636 \text{ kips}$$

$$\phi R_n = 636 \text{ kips}$$

Right side:



$$A_{gv} = 2(11)t_w = 2(11)(0.44) = 9.68 \text{ in}^2$$

$$A_{nv} = A_{gv} - 2(3.5)d_h t_w = 9.68 - 2(3.5)(1)(0.44) = 6.6 \text{ in}^2$$

$$A_{gt} = 6t_w = 2.64 \text{ in}^2$$

$$A_{nt} = A_{gt} - 2d_h t_w = 1.76 \text{ in}^2$$

$$R_n = 0.6(65)(6.6) + 65(1.76) \leq 0.6(50)(9.68) + 65(1.76)$$

$$R_n = 371.8 \leq 404.8$$

← governs

$$\phi R_n = 0.75(371.8) = 278.9 \text{ kips}$$

$$\phi R_n = \underline{279 \text{ kips}}$$

* Block Shear on the right side gussets

$$\phi R_n = 279 \text{ kips}$$

Problem 3

$$L = 15'$$

A992

$$F_y = 50 \text{ ksi}$$

$$F_u = 65 \text{ ksi}$$

$$P_u = 2008 \text{ kips}$$

W14 ?

$$k_y = 1.0$$

Part 1

not braced

$$k_x = 1.9$$

$$k_y = 1.0$$

$$(kL)_y = 15'$$

Load Col. Tables

$$\rightarrow \text{Try } W14 \times 176 \quad \phi_c P_n = 2010 \text{ kips}$$

$$\frac{r_x}{r_y} = 1.60$$

$$(kL)_{y, \text{equiv}} = \frac{(kL)_x}{r_x/r_y} = \frac{(1.9 \cdot 15)}{1.60} = 17.8$$

$$(kL)_{y, \text{equiv}} > (kL)_y \rightarrow x\text{-axis governs}$$

$$\text{re-enter w/ } (kL)_{y, \text{equiv}} = 17.8 \rightarrow \phi_c P_n = 1890 < 2008$$

$$\text{Try } W14 \times 193 \quad \phi_c P_n = 2070 \text{ kips} > 2008 \text{ kips} \quad \underline{\text{ok}}$$

Use W14 x 193.

Part 2

$$k_x = 0.76 \quad k_y = 1.0$$

$$\text{Load col. tables} \rightarrow \text{Try } W14 \times 176 \quad \phi_c P_n = 2010 \text{ kips}$$

$$\frac{r_x}{r_y} = 1.0$$

$$(kL)_{y, \text{equiv}} = \frac{(0.76 \cdot 15)}{1.0} = 7.125$$

$$(kL)_{y, \text{equiv}} < (kL)_y \rightarrow \text{assumption was correct.}$$

Use W14 x 176

Problem 4

$$(KL)_x = 30' \quad (KL)_y = 22'$$

$$A992 \quad F_y = 50 \text{ ksi} \\ F_u = 65 \text{ ksi} \\ E = 29000 \text{ ksi}$$

1) ϕP_n . Compactness.

$$A = A_{web} + A_{plates} = 2(22'')(2'') + 2(4.5'')(14'')$$

$$A = 214 \text{ in}^2$$

$$I_x = \frac{1}{12} (18'')(22'')^3 - \frac{1}{12} (14'')(13'')^3 \\ = 15972 - 2563.2 = 13409 \text{ in}^4$$

$$I_x = 13409 \text{ in}^4$$

$$I_y = \frac{1}{12} (22'')(18'')^3 - \frac{1}{12} (13'')(14'')^3 \\ = 10692 - 2973 = 7719 \text{ in}^4$$

$$I_y = 7719 \text{ in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{13409}{214}} = 7.92 \text{ in}$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{7719}{214}} = 6.00 \text{ in}$$

$$\left(\frac{KL}{r}\right)_x = \frac{30 \cdot 12}{7.92} = 45.5 \quad \rightarrow \text{governs; member will buckle about the x-axis}$$

$$\left(\frac{KL}{r}\right)_y = \frac{22 \cdot 12}{6.0} = 44$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)_x^2} = \frac{\pi^2 (29000)}{(45.5)^2} = 138.3$$

$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29000}{50}} = 113.4$$

$$\left(\frac{KL}{r}\right)_x = 45.5 < 4.71 \sqrt{\frac{E}{F_y}} = 113.4$$

$$\rightarrow F_{cr} = \left[0.658 \frac{F_y}{F_e} \right] F_y = \left[0.658 \frac{50}{138.4} \right] \cdot 50$$

$$F_{cr} = 42.98 \text{ ksi}$$

* can also use Table 4-22

$$\phi P_n = \phi F_u A_g = 0.9(42.98)(214) = 8277.9 \text{ kips}$$

$$\boxed{\phi P_n = 8278 \text{ kips}}$$

Check if section is compact :

$$\textcircled{1} \lambda = \frac{b}{t} = \frac{14''}{4.5''} = 3.11$$

$$\textcircled{2} \lambda = \frac{b}{t} = \frac{22''}{2''} = 11$$

Table B4.1 $\lambda_r = 1.49 \sqrt{\frac{E}{F_y}} = 1.49 \sqrt{\frac{29000}{50}} = 35.9$

Both λ are smaller than $\lambda_r = 35.9$

2) W14 x 730 ϕP_n

$$r_x = 8.17 \text{ in} \quad r_y = 4.69 \text{ in} \quad A_g = 215 \text{ in}^2$$

$$\left(\frac{KL}{r}\right)_x = \frac{(30 \cdot 12)}{8.17} = 44$$

$$\left(\frac{KL}{r}\right)_y = \frac{(22 \cdot 12)}{4.69} = 56.3 \rightarrow \text{governs; member will bend about y-axis}$$

From Table 4-1 $\boxed{\phi P_n = 7670 \text{ kips}}$

or $4.717 \sqrt{\frac{E}{F_y}} = 113.4 \quad \left(\frac{KL}{r}\right)_y = 4.717 \sqrt{\frac{E}{F_y}}$

$$F_{cr} = \left[0.658 \frac{F_c}{F_y}\right] \cdot F_y$$

$$F_c = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)_y^2} = 90.3 \text{ ksi} \rightarrow F_{cr} = 39.6 \text{ ksi}$$

$$\phi P_n = (0.9)(39.6)(215) = 7674 \text{ kips}$$

$$\boxed{\phi P_n = 7674 \text{ kips}}$$

Check compactness:

Table 1-1 says member is compact

$$\text{or } \lambda = \frac{bf}{2tf} = 1.82 < \lambda_r = 0.56 \sqrt{\frac{E}{F_y}} = 13.5 \quad \underline{\text{OK}}$$

$$\lambda = \frac{h}{t_w} = 3.71 < \lambda_r = 1.49 \sqrt{\frac{E}{F_y}} = 35.9 \quad \underline{\text{OK}}$$

3) Compare

The built up shape is stronger than the W-shape because it has a larger radius of gyration about the y-axis which makes for a larger I_y .

Due to increase values of I_y , the built-up shape is more difficult to buckle about the weak axis. (y-axis).