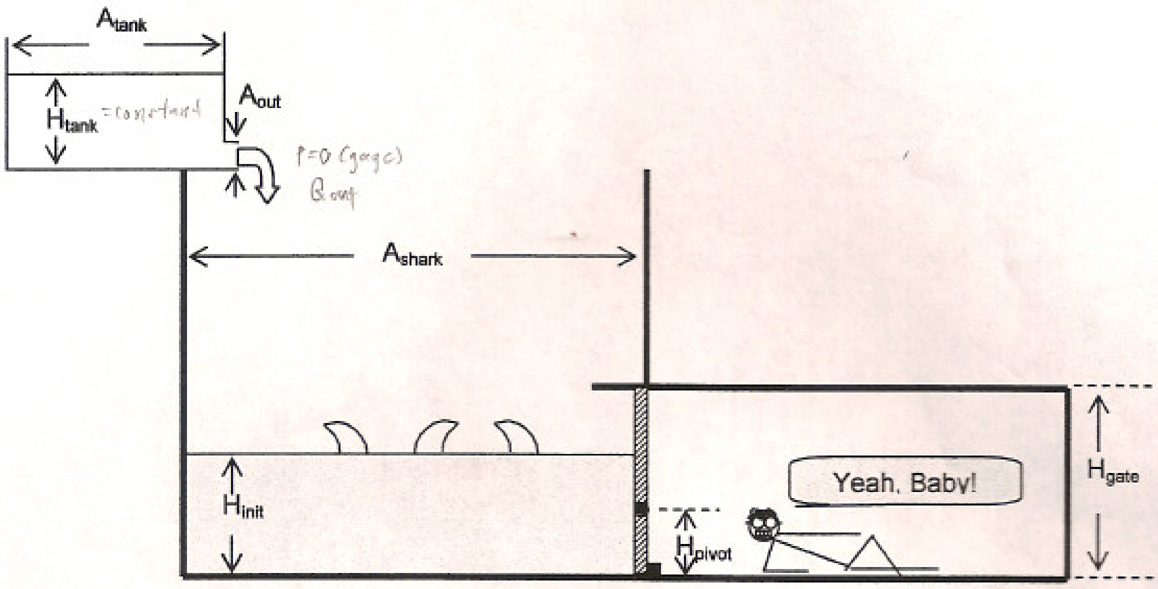
Hela Chan

Problem 1 (45 points):

Yeah, Baby!!! Dr. Evil has captured Austin Powers and is scheming to do him in at long last with his sharks with laser beams. Austin is currently held in a small chamber behind a gate held in place by hydrostatic forces. On the other side of the gate swim the sharks.

Above the shark tank, Dr. Evil has set up a second tank that is maintained at a constant depth of H_{tank} but drains out of an opening (as a free jet) to add water to the shark tank. As the level of water in the shark tank rises, at some point the gate will open, allowing the shark-infested water to flow into the chamber holding Austin Powers.



The question is: How long does Austin have to develop an escape plan? (that is, when will the gate open?)

Data Provided:

- H_{tank} = 50 cm
- A_{tank} = 4 m^2
- A_{out}: Circular outflow with radius of 10 cm.
- A_{shark} = 100 m^2
- H_{init} = 1 m
- H_{pivot} = 0.9 m
- H_{gate} = 2 m
- Width of Gate (into/out of page): 5 m
- Density of all fluids = 1030 kg/m^3

Bonus Question: Suppose that this whole set-up was on another planet where the gravitational acceleration was reduced from 9.8 m^2/s to 1 m^2/s. Would Austin have more time or less time to escape? Why?

PART 1:

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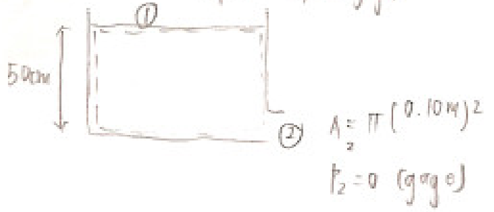


using Bernoulli to solve for V_2:

$$\cancel{\frac{P_1}{\gamma}}^{0} + \frac{V_1^2}{2g} + z_1 = \cancel{\frac{P_2}{\gamma}}^{0} + \frac{V_2^2}{2g} + z_2$$

from mass conservation

$$V_1 A_1 = V_2 A_2$$

$$V_1 = \frac{V_2 A_2}{A_1}$$

so

$$\frac{V_2^2\left(\frac{A_2}{A_1}\right)^2}{2g} + z_1 = \frac{V_2^2}{2g} + z_2$$

$$V_2^2\left[\left(\frac{A_2}{A_1}\right)^2 - 1\right] = 2g(z_2 - z_1)$$

$$V_2 = \sqrt{\frac{2g(z_2 - z_1)}{\left(\frac{A_2}{A_1}\right)^2 - 1}} = \sqrt{\frac{2(9.8\,m/s^2)(0 - 0.50m)}{\left[\frac{\pi(0.10)^2}{4m^2}\right]^2 - 1}}$$

$$V_2 = 3.13\ m/s$$

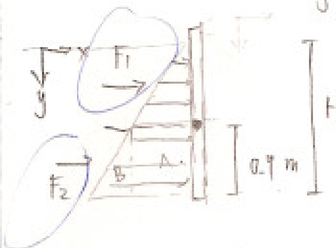
so Q outflow from upper tank is

$$Q_{out} = V_2 A_2 = (3.13\ m/s)[\pi(0.10m)^2]$$

$$Q_{out} = 9.835 \times 10^{-2}\ \frac{m^3}{s}$$

PART 2:

gate will open when moment caused by F_1 > moment caused by F_2 just use line of action



$F_1 = \int dp\,dA$

so $F_1 = \frac{1}{2}\gamma h h w = \frac{1}{2}(1030 \frac{kg}{m^3})(H-0.9)^2(5m) = 2575(H-0.9)^2$ N

$y_1 = \frac{1}{3}h = \frac{1}{3}(H-0.9)$ (from pivot point) moment arm

assumes $H \leq H_{gate}$ −5

using pressure prism method →

$$F_2 = \underbrace{\gamma h A}_{F_A} + \underbrace{\frac{1}{2}\gamma h_{pivot} A}_{F_B}$$

$$= [1030 \frac{kg}{m^2})(H-0.9)(0.9m \cdot 5m] + \frac{1}{2}(1030 \frac{kg}{m^2})(0.9m)(0.9m\ 5m)$$

$$= \underbrace{4633(H-0.9)}_{F_A} + \underbrace{2085.75}_{F_B}$$

$y_A = 0.45m$ (from pivot pt)

$y_B = \frac{2}{3}(0.9m)$ (from pivot pt)

$$\Sigma M_{pivot} = 0 \,(+\circlearrowleft) = -F_1 y_1 + F_B y_B + F_A y_A$$

$$0 = -2575(H-0.9)^2 \frac{1}{3}(H-0.9) + 2085.75\left(\frac{2}{3} \cdot 0.9\right) + 4633(H-0.9)(0.45)$$

setup ok except

H = ____ (stump by math ˙˙̂)

after finding H and given Qoutflow from top tank, the time can be found:

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$$Q_{outflow} = \frac{A_{tank}(H - H_{initial})}{t}$$

$$t = \frac{A_{tank}(H - H_{initial})}{Q_{outflow}} = \frac{(\cancel{4}\,100\,m^2)(H - 1m)}{9.835 \times 10^{-2}\,m^3/s}$$

-2

Bonus question - Austin would have more time because Qout is dependent on g, a smaller g yields a smaller Q out flow, which allows for a longer ~~greater~~ time before gate opens.

+2

This can be seen from the equation

$$t = \frac{A_{tank}(H - H_{initial})}{Q_{outflow}}.$$

yes.

the other values are not affected by g.

$F = \int dp\, dA$

$F_1 = \int \gamma y w\, dy$

$F_1 = \gamma w \frac{y^2}{2} = (1430\frac{kg}{m^3})(5m)\frac{(H - 0.9)^2}{2}$

$y_1 = \frac{1}{3}(H - 0.9)$

$F_2 = w\int_{H-0.9}^{H}(\gamma(H-0.9) + \gamma y)\, dy$

$= w\left(\gamma(H-0.9)y - \frac{\gamma y^2}{2}\right)_{H-0.9}^{H}$

$= w($

40/45

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$F = \int \gamma p\, dA$

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$= w\left(\gamma(H-0.9)y - \frac{\gamma y^2}{2}\right)\Big|_{H-0.9}^{H}$

$= w($

40/45

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Problem 2 (5 points):

Explain the difference between the local and convective accelerations. Why do we need to include both in our analysis of accelerations in fluid motions?

$$\frac{DV}{DT} = a = \underbrace{\frac{\partial V}{\partial t}}_{\text{local acceleration}} + \underbrace{u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z}}_{\text{convective acceleration}}$$

local acceleration is the change in velocity in time, so it is time dependent.

convective acceleration is the change in ~~velocity~~ in space (x, y, z), so it is not time dependent but depends on where particle is (location of fluid)

we need to include both in our analysis because the total change in velocity (acceleration) ~~felt~~ undergone by a fluid particle depends on both local and convective acceleration as shown by the equation above.

5/5